Acknowledgements

The author thanks the original authors of the Openstax modules on which this text is based and the Openstax project and support team. I also acknowledge the assistance of my pilot instructors at Wright College, Exodus Pettus and Hanna Krajewska, and their students. I further want to thank all of the adult education mathematics teachers at the City Colleges of Chicago who have provided suggestions and corrections. Finally, I wish to thank Rachel McCartney for the creation of numerous diagrams not found in the original modules and her assistance in entering, formatting, and proof reading the text. Any of the errors and ambiguities that remain are my fault alone.

Fred Siegeltuch
August 2019

How to Learn from a Math Textbook

A math textbook is different from other types of books. Here are tips on how to learn math from your math textbook.

Eliminate Distractions
Eliminate distractions so you can concentrate on the math. A quiet location helps. Soft music without words is OK and may help drown out other noises.

Slow Down
If you are reading a novel and are somewhat distracted, you can still get the story. When you are not concentrating on math, you will get very little out of it, and it will seem more difficult that it really is.

Every Word Counts
Extra words get in the way of clarity. Each page assumes you have mastered the previous pages. If lost, it is usually better to go back rather than forward. Math is logical and each step is necessary.

Understand Each Paragraph
Understand each paragraph before you go on. Reread as necessary for you to master an idea. Reading a math textbook can be difficult. It could take you half an hour to understand just one page.

Diagrams are Important
Diagrams and other illustrations are important. They are part of the written text. Stick with them until you thoroughly understand their content.

Write as you read!

> Work out sample problems. Rewrite each problem in your own words. Fill in all steps to clarify your understanding. When you go back and review, the steps are already filled in so you will continue to understand how each step was completed.
> Starting with the second example of each concept in the text, work the example. Do not worry if you cannot work the example at this time. After you have tried the example on your own, read the solution.

After reading an example, cover it up and try to work it out yourself. Continue rewriting and working the example until you can do it without the aid of the text.

Highlight formulas, definitions, cautionary notes (with an asterisk, check mark, etc.) in a way that is consistent. Do not overuse marking of the text.

Note any questions on concepts or procedures you need to have clarified.

Create a Cheat Sheet

Create a resource or “cheat” sheet by recording key points on a separate piece of paper or into a notebook.

Review

Review the day’s lesson after class and before the next class.

If you do not understand the material

Go back to the previous page and reread the information to maintain a train of thought.

Read the misunderstood paragraph aloud. Sometimes this helps you capture the meaning.

Consider watching a video on the subject. Some are suggested in the textbook.

Define exactly what you do not understand and call a study buddy for help.

Prepare questions on the confusing information and contact your math tutor or math instructor for help. Ask those questions at the next class meeting or contact the tutor or instructor by email.

Introduction
class="introduction"

Purchasing pounds of fruit at a fruit market requires a basic understanding of numbers. (credit: Dr. Karl-Heinz Hochhaus, Wikimedia Commons)

Even though counting is first taught at a young age, mastering mathematics, which is the study of numbers, requires constant attention. If it has been a while since you have studied math, it can be helpful to review basic topics. In this chapter, we will focus on numbers used for counting as well as four arithmetic operations—addition, subtraction, multiplication, and division.

We will also discuss some vocabulary that we will use throughout this book.

Introduction to Whole Numbers
By the end of this section, you will be able to:

- Contrast numbers, numerals, and digits
- Identify counting numbers and whole numbers
- Model whole numbers
- Identify the place value of a digit
- Use place value to name whole numbers
- Use place value to write whole numbers
- Round whole numbers

Contrast numbers, numerals, and digits

A **number** is a count or measure. It is an idea rather than a physical object.

A **numeral** is a name or symbol, or group of them, used to write or talk about numbers.

For example, when I want to refer to the idea of 5, I can write 5, or “five”, or if I’m using Roman numerals “V”. If I use a language other than English, the word or name for the idea 5 would be different. In Spanish, 5 is “cinco”. In Swahili, 5 is “tano”.

A **digit** is a single symbol used to refer to a number. The written text, 23, is a numeral, but it is not a digit because it uses two symbols, 2 and 3. 2 and 3 are each digits because they are single symbols that refer to a number.

Identify Counting Numbers and Whole Numbers

Learning algebra is similar to learning a language. You start with a basic vocabulary and then add to it as you go along. You need to practice often until the vocabulary becomes easy to you. The more you use the vocabulary, the more familiar it becomes.

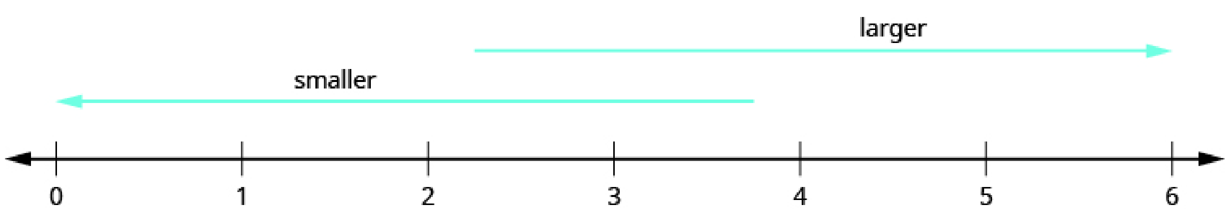
Algebra uses numbers and symbols to represent words and ideas. Let’s look at the numbers first. The most basic numbers used in algebra are those we use to count objects: $1, 2, 3, 4, 5, \ldots$ and so on. These are called the

counting numbers. The notation “…” is called an ellipsis, which is another way to show “and so on”, or that the pattern continues endlessly. Counting numbers are also called natural numbers.

Note:
Counting Numbers
The counting numbers start with 1 and continue.
Equation:

$$1, 2, 3, 4, 5 \ldots$$

Counting numbers and whole numbers can be visualized on a **number line** as shown in [link].



The numbers on the number line increase from left to right, and decrease from right to left.

The point labeled 0 is called the **origin**. The distance between the origin and the point labeled “1” is one unit. The next counting number is one more unit to the right. This continues as much as needed. When a number is paired with a point on the number line, it is called the **coordinate** of the point.

The discovery of the number zero was a big step in the history of mathematics. Including zero with the counting numbers gives a new set of numbers called the **whole numbers**.

Note:
Whole Numbers
The whole numbers are the counting numbers and zero.
Equation:

$$0, 1, 2, 3, 4, 5...$$

We stopped at 5 when listing the first few counting numbers and whole numbers. We could have written more numbers if they were needed to make the patterns clear.

Example:
Exercise:

Problem:

Which of the following are ⓐ counting numbers? ⓑ whole numbers?

$0, \frac{1}{4}, 3, 5.2, 15, -7, 105$

Solution:
Solution

- ⓐ The counting numbers start at 1, so 0 is not a counting number. The numbers $3, 15,$ and 105 are all counting numbers.
- ⓑ Whole numbers are counting numbers and 0. The numbers $0, 3, 15,$ and 105 are whole numbers.

The numbers $\frac{1}{4}$, -7, and 5.2 are neither counting numbers nor whole numbers. We will discuss these numbers later.

Note:
Exercise:

Problem:

Which of the following are ⓐ counting numbers ⓑ whole numbers?

$0, \frac{2}{3}, 2, 9, 11.8, 241, 376$

Solution:

- ⓐ 2, 9, 241, 376
- ⓑ 0, 2, 9, 241, 376

Note:
Exercise:

Problem:

Which of the following are ⓐ counting numbers ⓑ whole numbers?

$0, \frac{5}{3}, 7, 8.8, 13, -17, 201$

Solution:

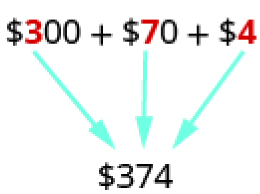
- ⓐ 7, 13, 201
- ⓑ 0, 7, 13, 201

Model Whole Numbers

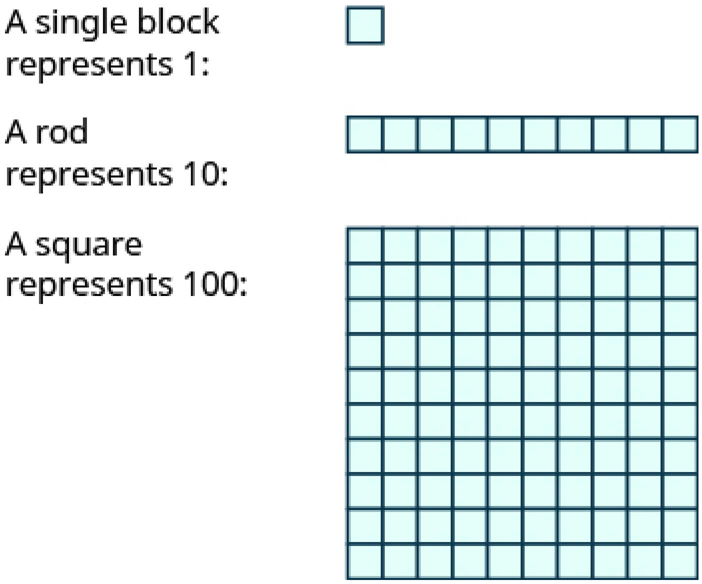
Our number system is called a **place value system** because the value of a digit depends on its position, or place, in a number. The number 537 has a different value than the number 735. Even though they use the same digits, their value is different because of the different placement of the 3 and the 7 and the 5.

Money gives us a familiar model of place value. Suppose a wallet contains three \$100 bills, seven \$10 bills, and four \$1 bills. The amounts are summarized in [link]. How much money is in the wallet?

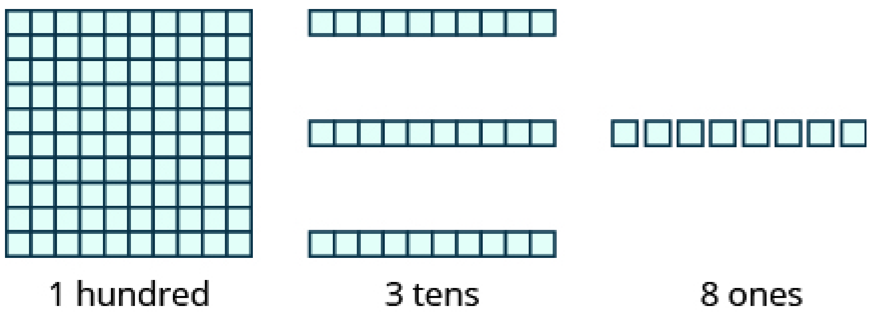
Find the total value of each kind of bill, and then add to find the total. The wallet contains \$374.



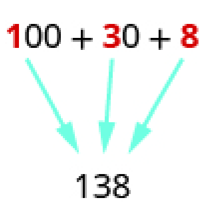
Base-10 blocks provide another way to model place value, as shown in [link]. The blocks can be used to represent hundreds, tens, and ones. Notice that the tens rod is made up of 10 ones, and the hundreds square is made of 10 tens, or 100 ones.



[link] shows the number 138 modeled with base-10 blocks.



We use place value notation to show the value of the number 138.



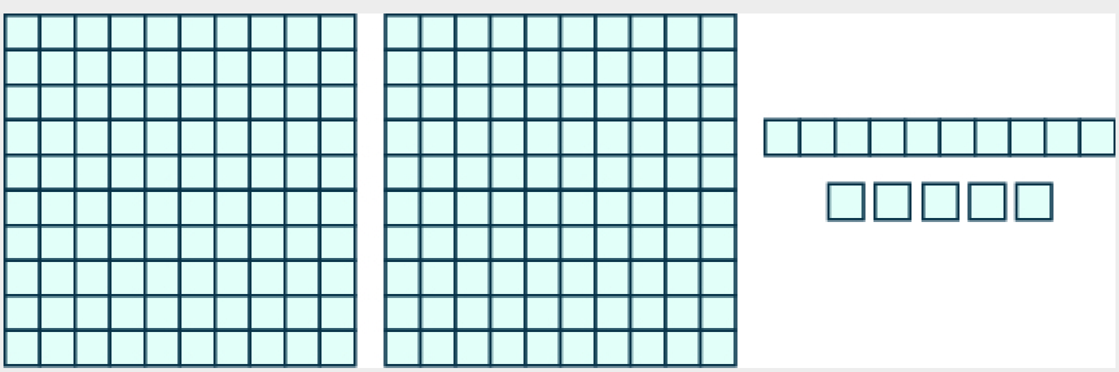
Digit	Place value	Number	Value	Total value
1	hundreds	1	100	100
3	tens	3	10	30
8	ones	8	1	$+\ 8$
				$\text{Sum} = 138$

Expanded Form
Both money and base-10 blocks illustrate the **expanded form** of a number. The expanded form explicitly shows the value of each digit.
The expanded form of 374 is 300 + 70 + 4, and the expanded form of 138 is 100 + 30 + 8.
Standard form, also called **compact form**, is the way we normally write numbers: 374 and 138.

Example:
Exercise:

Problem:

Use place value notation and expanded form to find the value of the number modeled by the base-10 blocks shown.



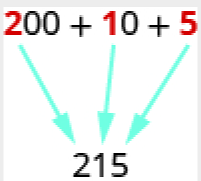
Solution:
Solution

There are 2 hundreds squares, which is 200.

There is 1 tens rod, which is 10.

There are 5 ones blocks, which is 5.

The expanded form is first followed by the standard form.



Digit	Place value	Number	Value	Total value
2	hundreds	2	100	200
1	tens	1	10	10
5	ones	5	1	$+\ 5$
				215

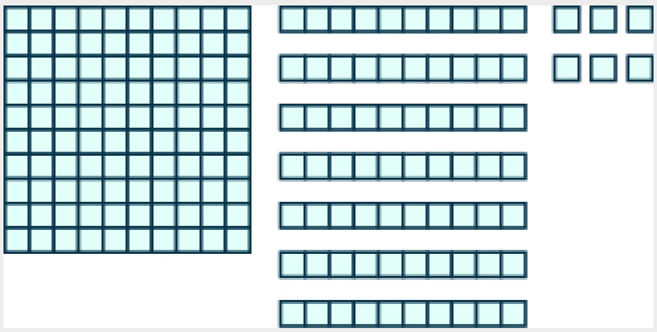
The base-10 blocks model the number 215.

Note:

Exercise:

Problem:

Use place value notation to find the expanded form and then the value of the number modeled by the base-10 blocks shown.



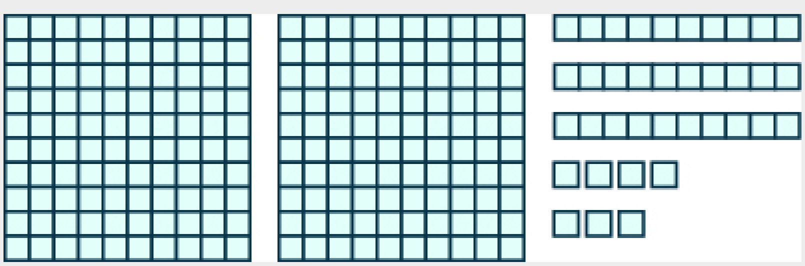
Solution:

Expanded form: 100 + 70 + 6
Value: 176

Note:
Exercise:

Problem:

Use place value notation to find expanded form and the value of the number modeled by the base-10 blocks shown.



Solution:

Expanded form: 200 + 30 + 7
Value: 237

Identify the Place Value of a Digit

By looking at money and base-10 blocks, we saw that each place in a number has a different value. A place value chart is a useful way to summarize this information. The place values are separated into groups of three, called periods. The periods are *ones*, *thousands*, *millions*, *billions*, *trillions*, and so on. In a written number, commas separate the periods.

Just as with the base-10 blocks, where the value of the tens rod is ten times the value of the ones block and the value of the hundreds square is ten times the tens rod, the value of each place in the place-value chart is ten times the value of the place to the right of it.

[link] shows how the number 5,278,194 is written in a place value chart.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

- The digit 5 is in the millions place. Its value is 5,000,000.
- The digit 2 is in the hundred thousands place. Its value is 200,000.
- The digit 7 is in the ten thousands place. Its value is 70,000.
- The digit 8 is in the thousands place. Its value is 8,000.
- The digit 1 is in the hundreds place. Its value is 100.
- The digit 9 is in the tens place. Its value is 90.
- The digit 4 is in the ones place. Its value is 4.

Example:
Exercise:

Problem:

In the number 63,407,218; find the place value of each of the following digits:

ⓐ 7
ⓑ 0
ⓒ 1
ⓓ 6
ⓔ 3

Solution:
Solution

Write the number in a place value chart, starting at the right.

Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

- ⓐ The 7 is in the thousands place.
- ⓑ The 0 is in the ten thousands place.
- ⓒ The 1 is in the tens place.
- ⓓ The 6 is in the ten millions place.
- ⓔ The 3 is in the millions place.

Note:
Exercise:

Problem:

For each number, find the place value of digits listed: 27,493,615

ⓐ 2

ⓑ 1
ⓒ 4
ⓓ 7
ⓔ 5

Solution:

ⓐ ten millions
ⓑ tens
ⓒ hundred thousands
ⓓ millions
ⓔ ones

Note:
Exercise:

Problem:

For each number, find the place value of digits listed:
519,711,641,328

ⓐ 9
ⓑ 4
ⓒ 2
ⓓ 6
ⓔ 7

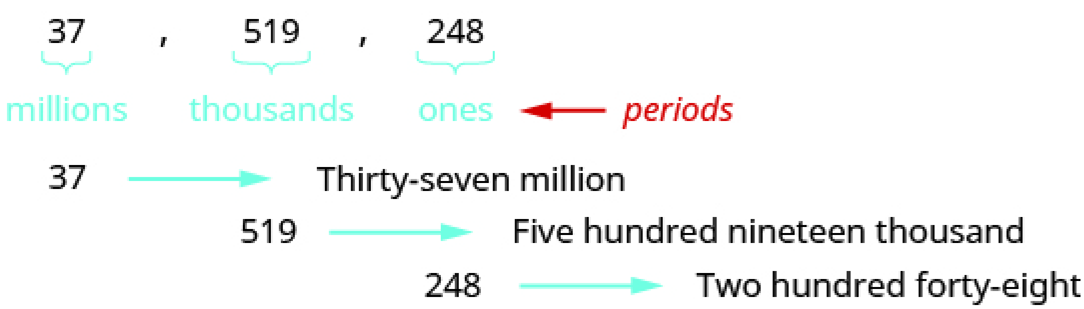
Solution:

ⓐ billions
ⓑ ten thousands
ⓒ tens
ⓓ hundred thousands

ⓔ hundred millions

Use Place Value to Name Whole Numbers

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period followed by the name of the period without the ‘s’ at the end. Start with the digit at the left, which has the largest place value. The commas separate the periods, so wherever there is a comma in the number, write a comma between the words. The ones period, which has the smallest place value, is not named.



So the number 37,519,248 is written thirty-seven million, five hundred nineteen thousand, two hundred forty-eight.

Notice that the word *and* is not used when naming a whole number.

Note:
Name a whole number in words.

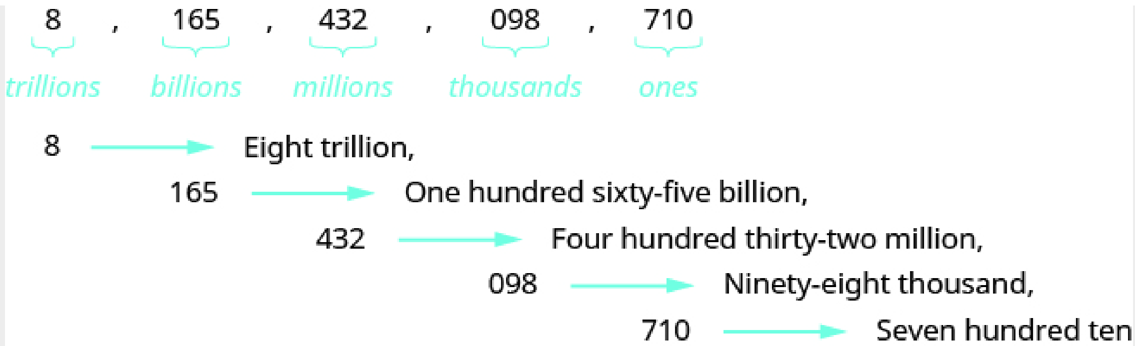
Starting at the digit on the left, name the number in each period, followed by the period name. Do not include the period name for the ones.
Use commas in the number to separate the periods.

Example:
Exercise:

Problem: Name the number 8,165,432,098,710 in words.

Solution:
Solution

Begin with the leftmost digit, which is 8. It is in the trillions place.	eight trillion
The next period to the right is billions.	one hundred sixty-five billion
The next period to the right is millions.	four hundred thirty-two million
The next period to the right is thousands.	ninety-eight thousand
The rightmost period shows the ones.	seven hundred ten



Putting all of the words together, we write 8,165,432,098,710 as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

A common error is to put the word "and" into the written whole number. For example, eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred and ten. Even though most people will know what number you are trying to write, it isn't correct.

Note:
Exercise:

Problem: Name each number in words: 9,258,137,904,061

Solution:

nine trillion, two hundred fifty-eight billion, one hundred thirty-seven million, nine hundred four thousand, sixty-one

Note:
Exercise:

Problem: Name each number in words: 17,864,325,619,004

Solution:

seventeen trillion, eight hundred sixty-four billion, three hundred twenty-five million, six hundred nineteen thousand, four

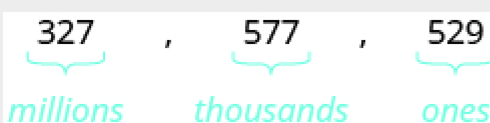
Example:
Exercise:

Problem:

A student conducted research and found that the number of mobile phone users in the United States during one month in 2014 was 327,577,529. Name that number in words.

Solution:
Solution

Identify the periods associated with the number.



Name the number in each period, followed by the period name. Put the commas in to separate the periods.

Millions period: three hundred twenty-seven million

Thousands period: five hundred seventy-seven thousand

Ones period: five hundred twenty-nine

So the number of mobile phone users in the Unites States during the month of April was three hundred twenty-seven million, five hundred seventy-seven thousand, five hundred twenty-nine.

Note:
Exercise:

Problem:

The population in a country is 316,128,839. Name that number.

Solution:

three hundred sixteen million, one hundred twenty-eight thousand, eight hundred thirty nine

Note:
Exercise:

Problem: One year is 31,536,000 seconds. Name that number.

Solution:

thirty one million, five hundred thirty-six thousand

Use Place Value to Write Whole Numbers

We will now reverse the process and write a number given in words as digits.

Note:
Use place value to write a whole number.

Identify the words that indicate periods. (Remember the ones period is never named.)

Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Name the number in each period and place the digits in the correct place value position.

Example:
Exercise:

Problem: Write the following numbers using digits.

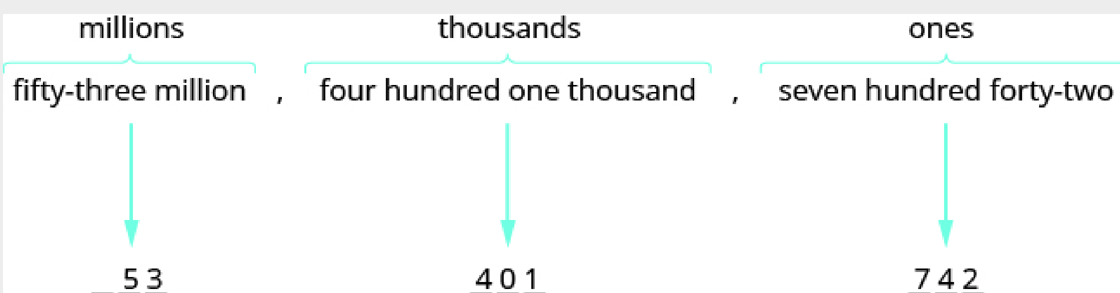
- ⓐ fifty-three million, four hundred one thousand, seven hundred forty-two
- ⓑ nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine

Solution:
Solution

ⓐ Identify the words that indicate periods.

Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.

Then write the digits in each period.



Put the numbers together, including the commas. The number is 53,401,742.

ⓑ Identify the words that indicate periods.

Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.

Then write the digits in each period.

The number is 9,246,073,189.

Notice that in part ⓑ, a zero was needed as a place-holder in the hundred thousands place. Be sure to write zeros as needed to make sure that each period, except possibly the first, has three places.

Note:
Exercise:

Problem: Write each number in standard form:

fifty-three million, eight hundred nine thousand, fifty-one.

Solution:

53,809,051

Note:
Exercise:

Problem: Write each number in standard form:

two billion, twenty-two million, seven hundred fourteen thousand, four hundred sixty-six.

Solution:

2,022,714,466

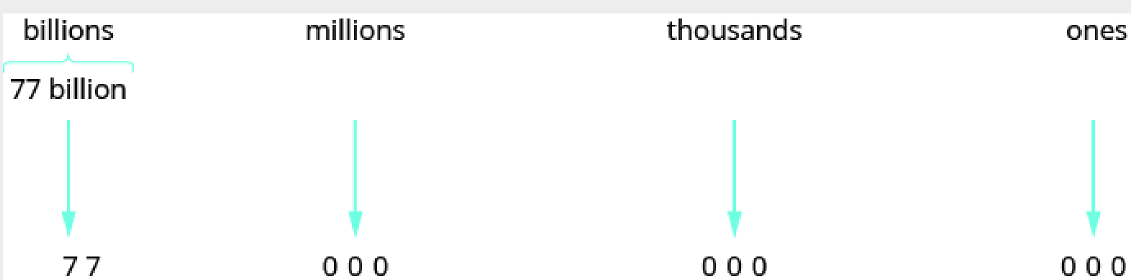
Example:
Exercise:

Problem:

A state budget was about $77 billion. Write the budget in standard form.

Solution:
Solution

Identify the periods. In this case, only two digits are given and they are in the billions period. To write the entire number, write zeros for all of the other periods.



So the budget was about $77,000,000,000.

Note:
Exercise:

Problem: Write each number in standard form:

The closest distance from Earth to Mars is about 34 million miles.

Solution:

34,000,000 miles

Note:
Exercise:

Problem: Write each number in standard form:

The total weight of an aircraft carrier is 204 million pounds.

Solution:

204,000,000 pounds

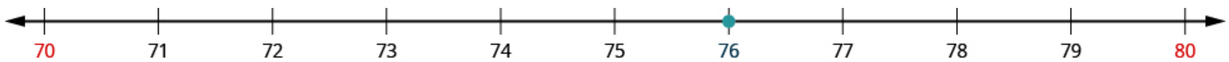
Round Whole Numbers

In 2018, the U.S. Census Bureau reported the population of the state of Illinois as 12,768,320 people. It might be enough to say that the population is approximately 13 million. The word *approximately* means that 13 million is not the exact population, but is close to the exact value.

The process of approximating a number is called **rounding**. Numbers are rounded to a specific place value depending on how much accuracy is needed. 13 million was achieved by rounding to the millions place. Had we

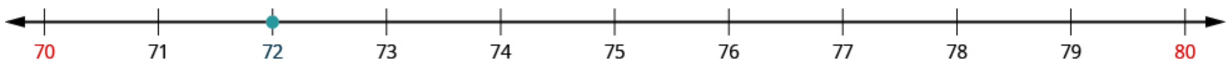
rounded to the one hundred thousands place, we would have 12,800,000 as a result. Had we rounded to the thousands place, we would have 12,768,000 as a result, and so on. The place value to which we round to depends on how we need to use the value.

Using the number line can help you visualize and understand the rounding process. Look at the number line in [link]. Suppose we want to round the number 76 to the nearest ten. Is 76 closer to 70 or 80 on the number line?



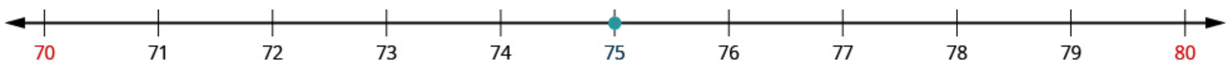
We can see that 76 is closer to 80 than to 70. So 76 rounded to the nearest ten is 80.

Now consider the number 72. Find 72 in [link].



We can see that 72 is closer to 70, so 72 rounded to the nearest ten is 70.

How do we round 75 to the nearest ten. Find 75 in [link].



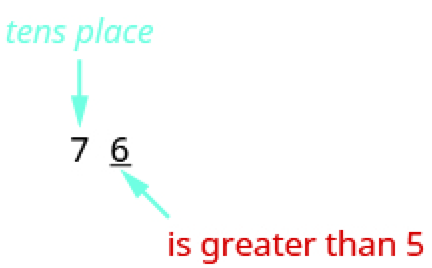
The number 75 is exactly midway between 70 and 80.

So that everyone rounds the same way in cases like this, mathematicians have agreed to round to the higher number, 80. So, 75 rounded to the nearest ten is 80.

Now that we have looked at this process on the number line, we can introduce a more general procedure. To round a number to a specific place, look at the number to the right of that place. If the number is less than 5, round down. If it is greater than or equal to 5, round up.

Notice that if mathematicians had decided to round 5 down rather than up, rounding would not be as easy. Consider the number 7,506 rounded to the thousands place. 7,500 is half way between 7,000 and 8,000 so 7,506 is closer to 8,000. Therefore 7,506 should be rounded to 8,000. Rounding this way, you can not tell if a number should be rounded up or down just by looking at the single digit to the right of the place that you are rounding to. But if we agree to always round up with a 5 then we can. This is because any digits to the right of the 5 can never make the value smaller.

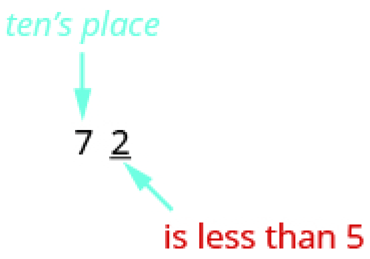
So, for example, to round 76 to the nearest ten, we look at the digit in the ones place.



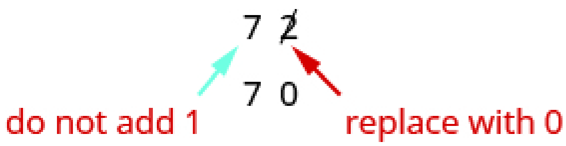
The digit in the ones place is a 6. Because 6 is greater than or equal to 5, we increase the digit in the tens place by one. So the 7 in the tens place becomes an 8. Now, replace any digits to the right of the 8 with zeros. So, 76 rounds to 80.

76 rounded to the nearest ten is 80.

Let's look again at rounding 72 to the nearest 10. Again, we look to the ones place.



The digit in the ones place is 2. Because 2 is less than 5, we keep the digit in the tens place the same and replace the digits to the right of it with zero. So 72 rounded to the nearest ten is 70.



Note:
Round a whole number to a specific place value.

Locate the given place value. All digits to the left of that place value do not change.
Underline the digit to the right of the given place value.
Determine if this digit is greater than or equal to 5.

- Yes—add 1 to the digit in the given place value.
- No—do not change the digit in the given place value.

Replace all digits to the right of the given place value with zeros.

Example:
Exercise:

Problem: Round 843 to the nearest ten.

Solution:
Solution

Locate the tens place.	
Underline the digit to the right of the tens place.	$84\underline{3}$
Since 3 is less than 5, do not change the digit in the tens place.	$84\underline{3}$
Replace all digits to the right of the tens place with zeros.	$84\underline{0}$
	Rounding 843 to the nearest ten gives 840.

Note:
Exercise:

Problem: Round to the nearest ten: 157.

Solution:

160

Note:
Exercise:

Problem: Round to the nearest ten: 884.

Solution:

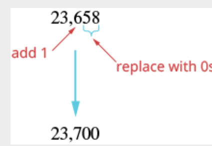
880

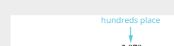
Example:
Exercise:

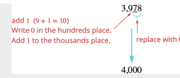
Problem: Round each number to the nearest hundred:

ⓐ $23,658$
ⓑ $3,978$

Solution:
Solution

ⓐ	
Locate the hundreds place.	
The digit of the right of the hundreds place is 5. Underline the digit to the right of the hundreds place.	23,6$\underline{5}$8
Since 5 is greater than or equal to 5, round up by adding 1 to the digit in the hundreds place. Then replace all digits to the right of the hundreds place with zeros.	 So 23,658 rounded to the nearest hundred is 23,700.

ⓑ	
Locate the hundreds place.	
Underline the digit to the right of the hundreds place.	3,9$\underline{7}$8
The digit to the right of the hundreds place is 7.	

Since 7 is greater than or equal to 5, round up by added 1 to the 9. Then place all digits to the right of the hundreds place with zeros.	 So 3,978 rounded to the nearest hundred is 4,000.

Note:
Exercise:

Problem: Round to the nearest hundred: 17,852.

Solution:

17,900

Note:
Exercise:

Problem: Round to the nearest hundred: 4,951.

Solution:

5,000

Example:

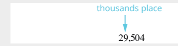
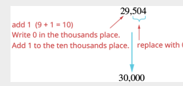
Exercise:

Problem: Round each number to the nearest thousand:

ⓐ 147,032
ⓑ 29,504

Solution:
Solution

ⓐ	
Locate the thousands place. Underline the digit to the right of the thousands place.	
The digit to the right of the thousands place is 0. Since 0 is less than 5, we do not change the digit in the thousands place.	147,032
We then replace all digits to the right of the thousands pace with zeros.	147,032 So 147,032 rounded to the nearest thousand is 147,000.

ⓑ	
Locate the thousands place.	
Underline the digit to the right of the thousands place.	29,504
The digit to the right of the thousands place is 5. Since 5 is greater than or equal to 5, round up by adding 1 to the 9. Then replace all digits to the right of the thousands place with zeros.	 So 29,504 rounded to the nearest thousand is 30,000.

Notice that in part ⓑ, when we add 1 thousand to the 9 thousands, the total is 10 thousands. We regroup this as 1 ten thousand and 0 thousands. We add the 1 ten thousand to the 3 ten thousands and put a 0 in the thousands place.

Note:
Exercise:

Problem: Round to the nearest thousand: 63,921.

Solution:

64,000

Note:
Exercise:

Problem: Round to the nearest thousand: $156,437$.

Solution:

156,000

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Determine Place Value
- Write a Whole Number in Digits from Words
- Contrast Numbers, Numerals, and Digits

Key Concepts

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

- **Name a whole number in words.**

  Starting at the digit on the left, name the number in each period, followed by the period name. Do not include the period name for the ones.
  Use commas in the number to separate the periods.

- **Use place value to write a whole number.**

  Identify the words that indicate periods. (Remember the ones period is never named.)
  Draw three blanks to indicate the number of places needed in each period.
  Name the number in each period and place the digits in the correct place value position.

- **Round a whole number to a specific place value.**

  Locate the given place value. All digits to the left of that place value do not change.
  Underline the digit to the right of the given place value.
  Determine if this digit is greater than or equal to 5. If yes—add 1 to the

digit in the given place value. If no—do not change the digit in the given place value.
Replace all digits to the right of the given place value with zeros.

Exercises

Practice Makes Perfect

Identify Counting Numbers and Whole Numbers

In the following exercises, determine which of the following numbers are ⓐ counting numbers ⓑ whole numbers.

Exercise:

Problem: $0, \frac{2}{3}, 5, 8.1, 125$

Solution:

ⓐ 5, 125
ⓑ 0, 5, 125

Exercise:

Problem: $0, \frac{7}{10}, 3, 20.5, 300$

Exercise:

Problem: $0, \frac{4}{9}, 3.9, 50, -120, 221$

Solution:

ⓐ 50, 221
ⓑ 0, 50, 221

Exercise:

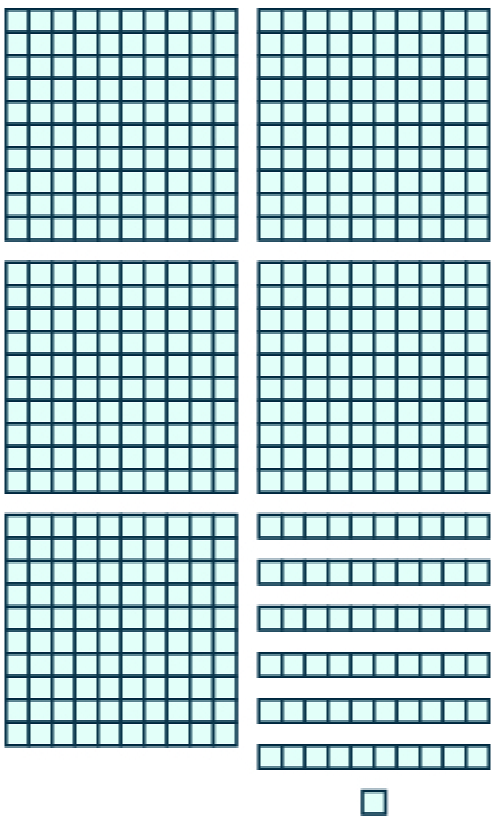
Problem: $0, \frac{3}{5}, 10, 303, 422.6$

Model Whole Numbers

In the following exercises, use place value notation to find the expanded form and the value of the number modeled by the base-10 blocks.

Exercise:

Problem:

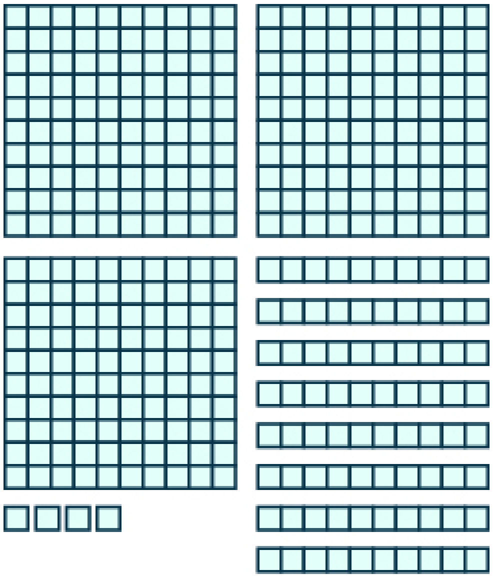


Solution:

Expanded form: 500 + 60 + 1
Value: 561

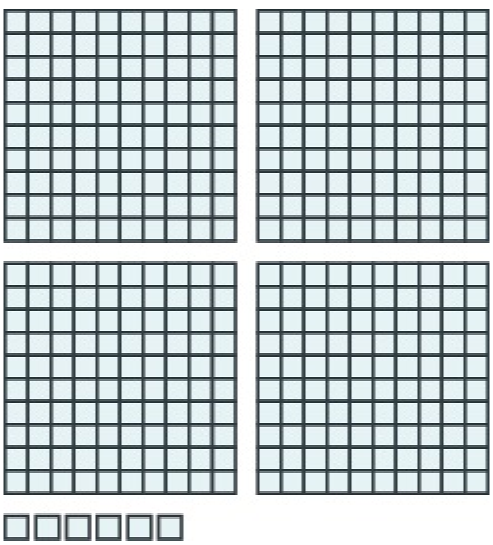
Exercise:

Problem:



Exercise:

Problem:

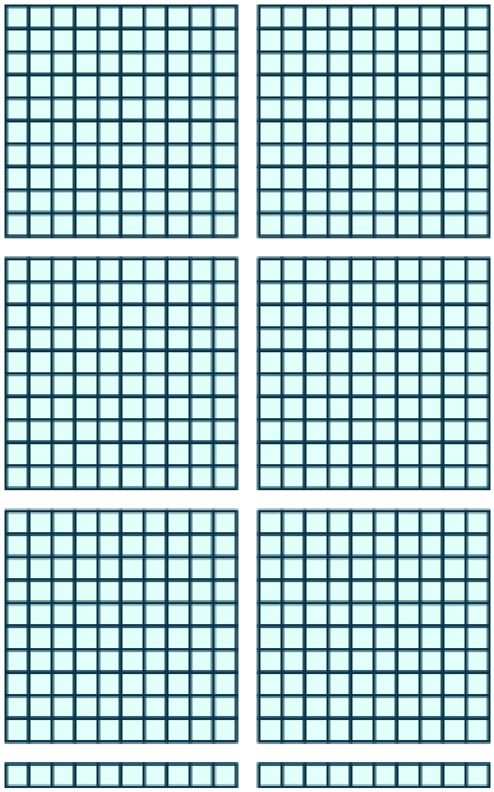


Solution:

Expanded form: 400 + 0 + 6
Value: 406

Exercise:

Problem:



Identify the Place Value of a Digit

In the following exercises, find the place value of the given digits.

Exercise:

Problem: 579,601

ⓐ 9
ⓑ 6
ⓒ 0
ⓓ 7
ⓔ 5

Solution:

ⓐ thousands
ⓑ hundreds

ⓒ tens
ⓓ ten thousands
ⓔ hundred thousands

Exercise:

Problem: 398,127

ⓐ 9
ⓑ 3
ⓒ 2
ⓓ 8
ⓔ 7

Exercise:

Problem: 56,804,379

ⓐ 8
ⓑ 6
ⓒ 4
ⓓ 7
ⓔ 0

Solution:

ⓐ hundred thousands
ⓑ millions
ⓒ thousands
ⓓ tens
ⓔ ten thousands

Exercise:

Problem: 78,320,465

ⓐ 8
ⓑ 4
ⓒ 2
ⓓ 6
ⓔ 7

Use Place Value to Name Whole Numbers

In the following exercises, name each number in words.

Exercise:

Problem: 1,078

Solution:

One thousand, seventy-eight

Exercise:

Problem: 5,902

Exercise:

Problem: 364,510

Solution:

Three hundred sixty-four thousand, five hundred ten

Exercise:

Problem: 146,023

Exercise:

Problem: 5,846,103

Solution:

Five million, eight hundred forty-six thousand, one hundred three

Exercise:

Problem: 1,458,398

Exercise:

Problem: 37,889,005

Solution:

Thirty seven million, eight hundred eighty-nine thousand, five

Exercise:

Problem: 62,008,465

Exercise:

Problem: The height of Mount Ranier is 14,410 feet.

Solution:

Fourteen thousand, four hundred ten

Exercise:

Problem: The height of Mount Adams is 12,276 feet.

Exercise:

Problem: Seventy years is 613,200 hours.

Solution:

Six hundred thirteen thousand, two hundred

Exercise:

Problem: One year is $525,600$ minutes.

Exercise:

Problem:

The U.S. Census estimate of the population of Miami-Dade county was $2,617,176$.

Solution:

Two million, six hundred seventeen thousand, one hundred seventy-six

Exercise:

Problem: The population of Chicago was $2,718,782$.

Exercise:

Problem:

There are projected to be $23,867,000$ college and university students in the US in five years.

Solution:

Twenty three million, eight hundred sixty-seven thousand

Exercise:

Problem:

About twelve years ago there were $20,665,415$ registered automobiles in California.

Exercise:

Problem:

The population of China is expected to reach $1,377,583,156$ in 2016.

Solution:

One billion, three hundred seventy-seven million, five hundred eighty-three thousand, one hundred fifty-six

Exercise:

Problem:

The population of India is estimated at $1,267,401,849$ as of July $1, 2014$.

Use Place Value to Write Whole Numbers

In the following exercises, write each number as a whole number using digits.

Exercise:

Problem: four hundred twelve

Solution:

412

Exercise:

Problem: two hundred fifty-three

Exercise:

Problem: thirty-five thousand, nine hundred seventy-five

Solution:

35,975

Exercise:

Problem: sixty-one thousand, four hundred fifteen

Exercise:

Problem:

eleven million, forty-four thousand, one hundred sixty-seven

Solution:

11,044,167

Exercise:

Problem:

eighteen million, one hundred two thousand, seven hundred eighty-three

Exercise:

Problem:

three billion, two hundred twenty-six million, five hundred twelve thousand, seventeen

---

Solution:

3,226,512,017

Exercise:

Problem:

eleven billion, four hundred seventy-one million, thirty-six thousand, one hundred six

Exercise:

Problem:

The population of the world was estimated to be seven billion, one hundred seventy-three million people.

Solution:

7,173,000,000

Exercise:

Problem:

The age of the solar system is estimated to be four billion, five hundred sixty-eight million years.

Exercise:

Problem:

Lake Tahoe has a capacity of thirty-nine trillion gallons of water.

Solution:

39,000,000,000,000

Exercise:

Problem:

The federal government budget was three trillion, five hundred billion dollars.

Round Whole Numbers

In the following exercises, round to the indicated place value.

Exercise:

Problem: Round to the nearest ten:

ⓐ 386
ⓑ 2,931

Solution:

ⓐ 390
ⓑ 2,930

Exercise:

Problem: Round to the nearest ten:

ⓐ 792
ⓑ 5,647

Exercise:

Problem: Round to the nearest hundred:

ⓐ 13,748
ⓑ 391,794

Solution:

ⓐ 13,700
ⓑ 391,800

Exercise:

Problem: Round to the nearest hundred:

ⓐ 28,166
ⓑ 481,628

Exercise:

Problem: Round to the nearest ten:

ⓐ 1,492
ⓑ 1,497

Solution:

ⓐ 1,490
ⓑ 1,500

Exercise:

Problem: Round to the nearest thousand:

ⓐ 2,391
ⓑ 2,795

Exercise:

Problem: Round to the nearest hundred:

ⓐ 63,994
ⓑ 63,949

Solution:

ⓐ 64,000
ⓑ 63,900

Exercise:

Problem: Round to the nearest thousand:

ⓐ 163,584
ⓑ 163,246

Everyday Math

Exercise:

Problem:

Writing a Check Jorge bought a car for $24,493. He paid for the car with a check. Write the purchase price in words.

Solution:

Twenty four thousand, four hundred ninety-three dollars

Exercise:

Problem:

Writing a Check Marissa’s kitchen remodeling cost $18,549. She wrote a check to the contractor. Write the amount paid in words.

Exercise:

Problem:

Buying a Car Jorge bought a car for $24,493. Round the price to the nearest:

ⓐ ten dollars
ⓑ hundred dollars
ⓒ thousand dollars
ⓓ ten-thousand dollars

Solution:

ⓐ $24,490
ⓑ $24,500
ⓒ $24,000
ⓓ $20,000

Exercise:

Problem:

Remodeling a Kitchen Marissa's kitchen remodeling cost $18,549. Round the cost to the nearest:

ⓐ ten dollars
ⓑ hundred dollars
ⓒ thousand dollars
ⓓ ten-thousand dollars

Exercise:

Problem:

Population The population of China was 1,355,692,544 in 2014. Round the population to the nearest:

ⓐ billion people
ⓑ hundred-million people
ⓒ million people

Solution:

ⓐ 1,000,000,000
ⓑ 1,400,000,000
ⓒ 1,356,000,000

Exercise:

Problem:

Astronomy The average distance between Earth and the sun is 149,597,888 kilometers. Round the distance to the nearest:

ⓐ hundred-million kilometers
ⓑ ten-million kilometers
ⓒ million kilometers

Writing Exercises

Exercise:

Problem:

In your own words, explain the difference between the counting numbers and the whole numbers.

Solution:

Answers may vary. The whole numbers are the counting numbers with the inclusion of zero.

Exercise:

Problem:

Give an example from your everyday life where it helps to round numbers.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
contrast numbers, numerals, and digits.			
identify counting numbers and whole numbers.			
model whole numbers.			
identify the place value of a digit.			
use place value to name whole numbers.			
use place value to write whole numbers.			
round whole numbers.			

ⓑ If most of your checks were...

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

compact form
: Same as standard form. The normal way we write whole numbers. Contrast with expanded form.

coordinate

A number paired with a point on a number line is called the coordinate of the point.

counting numbers

The counting numbers are the numbers 1, 2, 3, ….

expanded form

The expanded form explicitly shows the value of each digit.

number line

A number line is used to visualize numbers. The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left.

origin

The origin is the point labeled 0 on a number line.

place value system

Our number system is called a place value system because the value of a digit depends on its position, or place, in a number.

rounding

The process of approximating a number is called rounding.

standard form

Same as compact form.

whole numbers

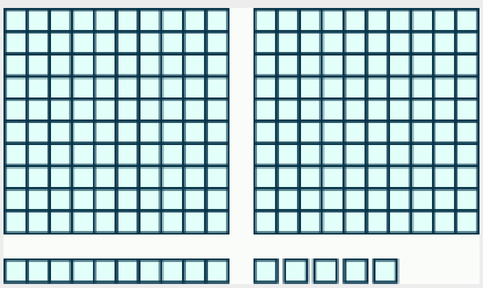
The whole numbers are the numbers 0, 1, 2, 3, ….

Add Whole Numbers
By the end of this section, you will be able to:

- Use addition notation
- Model addition of whole numbers
- Add whole numbers without models
- Explain when and how addition can be more efficient than counting
- Recognize and explain the commutative and associative properties of addition
- Translate word phrases to math notation
- Add whole numbers in applications

Note:
Before you get started, take this readiness quiz.

1. What is the number modeled by the base-10 blocks?



   If you missed this problem, review [link].
2. Write the number three hundred forty-two thousand six using digits.
   If you missed this problem, review [link].

Use Addition Notation

A college student has a part-time job. Last week he worked 3 hours on Monday and 4 hours on Friday. To find the total number of hours he worked last week, he added 3 and 4.

The operation of addition combines numbers to get a **sum**. The notation we use to find the sum of 3 and 4 is:

Equation:

$$3 + 4$$

We read this as *three plus four* and the result is the sum of three and four. The numbers 3 and 4 are called the **addends**. A math statement that includes numbers and operations is called an **expression**.

Note:
Addition Notation
To describe addition, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Addition	$+$	$3 + 4$	three plus four	the sum of 3 and 4

Example:
Exercise:

Problem: Translate from math notation to words:

ⓐ $7 + 1$
ⓑ $12 + 14$

Solution:
Solution

- ⓐ The expression consists of a plus symbol connecting the addends 7 and 1. We read this as *seven plus one*. The result is *the sum of seven and*

one.

- ⓑ The expression consists of a plus symbol connecting the addends 12 and 14. We read this as *twelve plus fourteen*. The result is the *sum of twelve and fourteen*.

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $8+4$
ⓑ $18+11$

Solution:

- ⓐ eight plus four; the sum of eight and four
- ⓑ eighteen plus eleven; the sum of eighteen and eleven

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $21+16$
ⓑ $100+200$

Solution:

ⓐ twenty-one plus sixteen; the sum of twenty-one and sixteen
ⓑ one hundred plus two hundred; the sum of one hundred and two hundred

Counting versus Adding

Counting and adding are both used to determine an amount. Counting a small amount normally takes a short time, but counting a large amount can take a long time. For example, counting the number of students in your classroom probably takes at most a minute while counting the number of students in the entire building will take much longer.

Addition can be more efficient than counting but that requires that the addends are already known. If I already have the number of students in each class, then it will be more efficient to add those values to find the number of students in the entire building than it would be to count all of the students directly. Getting those original values could have been done by previous counting, and adding makes use of that work. Simply recounting all of the students would be inefficient.

In order for addition to be most useful, one needs to know the addition facts and how to add.

Model Addition of Whole Numbers

Addition is often used to determine the amount after we put two different groups together. We will model addition with base-10 blocks. Remember, a block represents 1 and a rod represents 10. Let's start by modeling the addition expression we just considered, $3 + 4$.

Each addend is less than 10, so we can use ones blocks.

We start by modeling the first number with 3 blocks.	
Then we model the second number with 4 blocks.	

Put the parts together and count the total number of blocks.	

There are 7 blocks in all. We use an equal sign $(=)$ to show the sum. A math sentence that shows that two expressions are equal is called an equation. We have shown that. $3+4=7$.

Most people don’t need to count the total number of blocks to know that there are seven of them. They know their addition facts and automatically recall that 3 + 4 = 7. When numbers are small like 3 and 4, it doesn’t save much time to add rather than count. For large numbers, it makes a big difference.

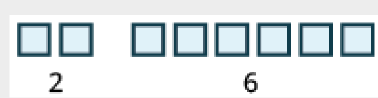
Example:
Exercise:

Problem: Model the addition $2+6$.

Solution:
Solution

$2+6$ means the sum of 2 and 6

Each addend is less than 10, so we can use ones blocks.

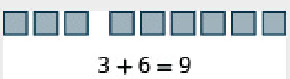
Model the first number with 2 blocks.	
Model the second number with 6 blocks.	

Put the parts together and count the total number of blocks	 There are 8 blocks in all, so $2 + 6 = 8$.

Note:
Exercise:

Problem: Model: $3 + 6$.

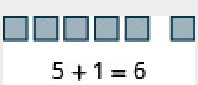
Solution:



Note:
Exercise:

Problem: Model: $5 + 1$.

Solution:



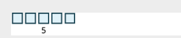
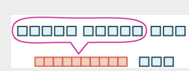
When the result is 10 or more ones blocks, we will exchange the 10 unit blocks for one rod.

Example:
Exercise:

Problem: Model the addition $5 + 8$.

Solution:
Solution

$5 + 8$ means the sum of 5 and 8.

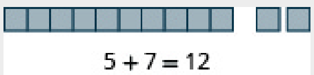
Each addend is less than 10, se we can use ones blocks.	
Model the first number with 5 blocks.	
Model the second number with 8 blocks.	
Count the result. There are more than 10 blocks so we exchange 10 ones blocks for 1 tens rod. We want to use the minimum number of pieces possible.	
Now we have 1 ten and 3 ones, which is 13.	5 + 8 = 13

Notice that we can describe the models as ones blocks and tens rods, or we can simply say *ones* and *tens*. From now on, we will use the shorter version but keep in mind that they mean the same thing.

Note:
Exercise:

Problem: Model the addition: $5 + 7$.

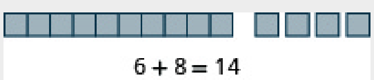
Solution:



Note:
Exercise:

Problem: Model the addition: $6 + 8$.

Solution:



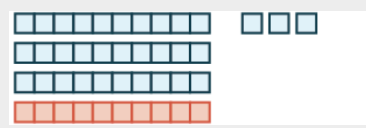
Next we will model adding two digit numbers.

Example:
Exercise:

Problem: Model the addition: $17 + 26$.

Solution:
Solution

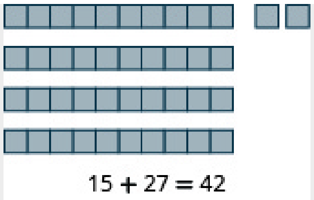
$17 + 26$ means the sum of 17 and 26.

Model the 17.	1 ten and 7 ones	
Model the 26.	2 tens and 6 ones	
Combine.	3 tens and 13 ones	
Exchange 10 ones for 1 ten.	4 tens and 3 ones $40 + 3 = 43$	
We have shown that $17 + 26 = 43$		

Note:
Exercise:

Problem: Model each addition: $15 + 27$.

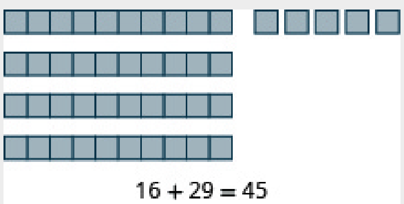
Solution:



Note:
Exercise:

Problem: Model each addition: $16 + 29$.

Solution:



Add Whole Numbers Without Models

Now that we have used models to add numbers, we can move on to adding without models. Before we do that, make sure you know all the one digit addition facts. You will need to use these number facts when you add larger numbers.

Imagine filling in [link] by adding each row number along the left side to each column number across the top. Make sure that you get each sum shown. If you have trouble, model it. It is important that you memorize any number facts you do not already know so that you can quickly and reliably use the number facts when you add larger numbers.

+	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Did you notice what happens when you add zero to a number? The sum of any number and zero is the number itself. We call this the Identity Property of Addition. Zero is called the additive identity.

Note:
Identity Property of Addition
The sum of any number a and 0 is the number.
Equation:

$$a + 0 = a$$
$$0 + a = a$$

Frequently in algebra, we use a letter to stand for potentially any number. When used that way, that letter is a **variable**.

Each of these equations has only one variable, the letter “a”. The “a” appears twice in each equation. For either of these equations, once we pick a value for “a” we have to use the same value for “a” everywhere in the equation. If we wanted to be able to pick a different value for the second instance of the variable, we would have had to use a different variable, perhaps the letter “b” although any other letter would do.

Example:
Exercise:

Problem:

Find each sum and explain how the Identity Property of Addition applies:

ⓐ $0+11$
ⓑ $42+0$

Solution:
Solution

ⓐ The first addend is zero. The sum of zero and any number is the number. In the equation 0 + a = a, if a = 11 then 0 + 11 = 11. Notice that a = 11 in both places.	$0+11=11$
ⓑ The second addend is zero. The sum of any number and zero is the number. In the equation a + 0 = a, if a = 42 then 42 + 0 = 42. Notice that a = 42 in both places.	$42+0=42$

Note:
Exercise:

Problem: Find each sum:

ⓐ $0 + 19$
ⓑ $39 + 0$

Solution:

ⓐ $0 + 19 = 19$
ⓑ $39 + 0 = 39$

Note:
Exercise:

Problem: Find each sum:

ⓐ $0 + 24$
ⓑ $57 + 0$

Solution:

ⓐ $0 + 24 = 24$
ⓑ $57 + 0 = 57$

Look at the pairs of sums.

$2+3=5$	$3+2=5$
$4+7=11$	$7+4=11$
$8+9=17$	$9+8=17$

Notice that when the order of the addends is reversed, the sum does not change. This property is called the Commutative Property of Addition, which states that changing the order of the addends does not change their sum. Notice that this property requires two variables.

Note:
Commutative Property of Addition
Changing the order of the addends a and b does not change their sum.
Equation:

$$a+b=b+a$$

Example:
Exercise:

Problem: Add:

ⓐ $8+7$
ⓑ $7+8$

Solution:
Solution

ⓐ	

Add.	$8+7$
	15

ⓑ	
Add.	$7+8$
	15

Did you notice that changing the order of the addends did not change their sum? We could have immediately known the sum from part ⓑ just by recognizing that the addends were the same as in part ⓑ, but in the reverse order. As a result, both sums are the same.

An equation that shows this instance of the Commutative Property of Addition is 8 + 7 = 7 + 8.

Note:
Exercise:

Problem: Add: $9+7$ and $7+9$.

Solution:

$9+7=16; 7+9=16$

Note:
Exercise:

Problem: Add: $8 + 6$ and $6 + 8$.

Solution:

$8 + 6 = 14; 6 + 8 = 14$

Example:
Exercise:

Problem: Add: $28 + 61$.

Solution:
Solution

To add numbers with more than one digit, it is often easier to write the numbers vertically in columns.

Write the numbers so the ones and tens digits line up vertically.	$\begin{array}{r} 28 \\ +61 \\ \hline \end{array}$
Then add the digits in each place value. Add the ones: $8 + 1 = 9$ Add the tens: $2 + 6 = 8$	$\begin{array}{r} 28 \\ +61 \\ \hline 89 \end{array}$

Summing 28 with 61 was easy. We did not need any addition facts beyond the single digit facts even though both of our addends were greater than 9. We were able to add each place separately. Without this, we would have too many addition facts to learn and we might be better off counting. By grouping large numbers using place value, we can use the same single digit facts repeatedly

so that there is only a small number of facts to learn. This helps adding be much more efficient than counting.

Why do we ever bother to count rather than add? Adding requires that we know the addends. In a textbook problem, the addends are usually given, but in everyday life, we are often not that lucky. Counting may be the only way to find the addends.

Note:
Exercise:

Problem: Add: $32 + 54$.

Solution:

$32 + 54 = 86$

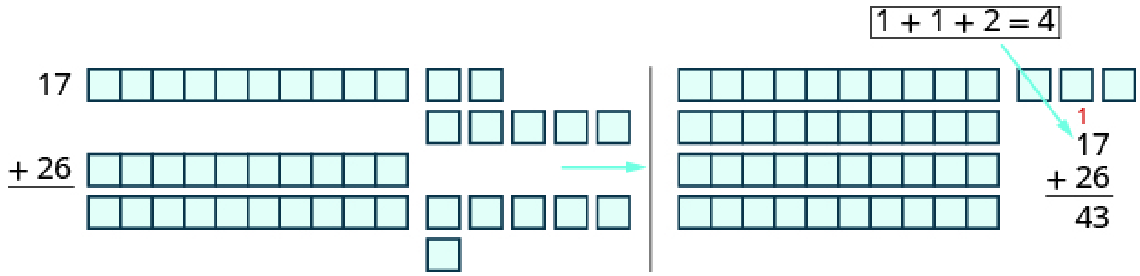
Note:
Exercise:

Problem: Add: $25 + 74$.

Solution:

$25 + 74 = 99$

In the previous example, the sum of the ones and the sum of the tens were both less than 10. But what happens if the sum is 10 or more? Let’s use our base-10 model to find out. [link] shows the addition of 17 and 26 again.



When we add the ones, $7 + 6$, we get 13 ones. Because we have more than 10 ones, we can exchange 10 of the ones for 1 ten. Now we have 4 tens and 3 ones. Without using the model, we show this as a small red 1 above the digits in the tens place.

When the sum in a place value column is greater than 9, we carry over to the next column to the left. Carrying is the same as regrouping by exchanging. For example, 10 ones for 1 ten or 10 tens for 1 hundred.

Note:
Add whole numbers.

Write the numbers so each place value lines up vertically.
Add the digits in each place value. Work from right to left starting with the ones place. If a sum in a place value is more than 9, carry to the next place value.
Continue adding each place value from right to left, adding each place value and carrying if needed.

Why do we add by starting with on the right (the ones’ place) and then move place by place to the left? Why not the other way around? The digits to the left are worth more than the digits to the right. Intuitively, is makes sense to start on the left side. When an approximate answer is likely to be all we need, this may be good enough. This is similar to rounding. If we need an exact answer, consider 17 + 26 again. If we add the tens place digits, we get 1 + 2 = 3. If we add the ones place digits, we get 7 + 6 = 13. If we have already written down the 3 in the tens’ place, we would have to change it to a 4 because of the regrouping (carrying) from the ones’ place. This is inefficient, and that is why it is better to start at the ones’ place and move to the left.

Example:
Exercise:

Problem: Add: $43 + 69$.

Solution:
Solution

Write the numbers so the digits line up vertically.	$\begin{array}{r} 43 \\ +69 \\ \hline \end{array}$
Add the digits in each place. Add the ones: $3 + 9 = 12$	
Write the 2 in the ones place in the sum. Add the 1 ten to the tens place.	$\begin{array}{r} \overset{1}{4}3 \\ +69 \\ \hline 2 \end{array}$
Now add the tens: $1 + 4 + 6 = 11$ Write the 11 in the sum.	$\begin{array}{r} \overset{1}{4}3 \\ +69 \\ \hline 112 \end{array}$

Note:
Exercise:

Problem: Add: $35 + 98$.

Solution:

$35 + 98 = 133$

Note:
Exercise:

Problem: Add: $72 + 89$.

Solution:

$72 + 89 = 161$

Example:
Exercise:

Problem: Add: $324 + 586$.

Solution:
Solution

Write the numbers so the digits line up vertically.	$\begin{array}{r} 324 \\ +\ 586 \\ \hline \end{array}$
Add the digits in each place value. Add the ones: $4 + 6 = 10$ Write the 0 in the ones place in the sum and carry the 1 ten to the tens place.	$\begin{array}{r} 3\overset{1}{2}4 \\ +\ 586 \\ \hline 0 \end{array}$
Add the tens: $1 + 2 + 8 = 11$ Write the 1 in the tens place in the sum and carry the 1 hundred to the hundreds	$\begin{array}{r} 3\overset{1}{2}4 \\ +\ 586 \\ \hline 0 \end{array}$

Add the hundreds: $1 + 3 + 5 = 9$ Write the 9 in the hundreds place.	$\begin{array}{r} \overset{}{3}\overset{1}{2}4 \\ +\ 586 \\ \hline 0 \end{array}$

Note:
Exercise:

Problem: Add: $456 + 376$.

Solution:

$456 + 376 = 832$

Note:
Exercise:

Problem: Add: $269 + 578$.

Solution:

$269 + 578 = 847$

Example:
Exercise:

Problem: Add: $1{,}683 + 479$.

Solution:
Solution

Write the numbers so the digits line up vertically.	$\begin{array}{r} 1{,}683 \\ +\ 479 \\ \hline \end{array}$
Add the digits in each place value.	
Add the ones: $3 + 9 = 12$. Write the 2 in the ones place of the sum and carry the 1 ten to the tens place.	$\begin{array}{r} {\scriptstyle 1\ } \\ 1{,}683 \\ +\ 479 \\ \hline 2 \end{array}$
Add the tens: $1 + 7 + 8 = 16$ Write the 6 in the tens place and carry the 1 hundred to the hundreds place.	$\begin{array}{r} {\scriptstyle 1\,1\ } \\ 1{,}683 \\ +\ 479 \\ \hline 62 \end{array}$
Add the hundreds: $1 + 6 + 4 = 11$ Write the 1 in the hundreds place and carry the 1 thousand to the thousands place.	$\begin{array}{r} {\scriptstyle 1\,1\ } \\ 1{,}683 \\ +\ 479 \\ \hline 162 \end{array}$
Add the thousands $1 + 1 = 2$. Write the 2 in the thousands place of the sum.	$\begin{array}{r} {\scriptstyle 1\,1\ } \\ 1{,}683 \\ +\ 479 \\ \hline 2{,}162 \end{array}$

When the addends have different numbers of digits, be careful to line up the corresponding place values starting with the ones and moving toward the left.

Note:
Exercise:

Problem: Add: $4{,}597 + 685$.

Solution:

$4{,}597 + 685 = 5{,}282$

Note:
Exercise:

Problem: Add: $5{,}837 + 695$.

Solution:

$5{,}837 + 695 = 6{,}532$

Example:
Exercise:

Problem: Add: $21{,}357 + 861 + 8{,}596$.

Solution:
Solution

Write the numbers so the place values line up vertically.	$\begin{array}{r} 21{,}357 \\ 861 \\ \underline{+\ 8{,}596} \end{array}$
Add the digits in each place value.	
Add the ones: $7 + 1 + 6 = 14$ Write the 4 in the ones place of the sum and carry the 1 to the tens place.	

	$\begin{array}{r} 21{,}3\overset{1}{5}7 \\ 861 \\ +\ 8{,}596 \\ \hline 4 \end{array}$
Add the tens: $1+5+6+9=21$ Write the 1 in the tens place and carry the 2 to the hundreds place.	$\begin{array}{r} 21{,}\overset{2}{3}\overset{1}{5}7 \\ 861 \\ +\ 8{,}596 \\ \hline 14 \end{array}$
Add the hundreds: $2+3+8+5=18$ Write the 8 in the hundreds place and carry the 1 to the thousands place.	$\begin{array}{r} 2\overset{1}{1}{,}\overset{2}{3}\overset{1}{5}7 \\ 861 \\ +\ 8{,}596 \\ \hline 814 \end{array}$
Add the thousands $1+1+8=10$. Write the 0 in the thousands place and carry the 1 to the ten thousands place.	$\begin{array}{r} \overset{1}{2}\overset{1}{1}{,}\overset{2}{3}\overset{1}{5}7 \\ 861 \\ +\ 8{,}596 \\ \hline 0814 \end{array}$
Add the ten-thousands $1+2=3$. Write the 3 in the ten thousands place in the sum.	$\begin{array}{r} \overset{1}{2}\overset{1}{1}{,}\overset{2}{3}\overset{1}{5}7 \\ 861 \\ +\ 8{,}596 \\ \hline 30{,}814 \end{array}$

This example had three addends. We can add any number of addends using the same process as long as we are careful to line up the place values correctly.

Note:
Exercise:

Problem: Add: $46{,}195 + 397 + 6{,}281$.

Solution:

$46{,}195 + 397 + 6{,}281 = 52{,}873$

Note:
Exercise:

Problem: Add: $53{,}762 + 196 + 7{,}458$.

Solution:

$53{,}762 + 196 + 7{,}458 = 61{,}416$

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation into words. Now we'll reverse the process. We'll translate word phrases into math notation. Some of the word phrases that indicate addition are listed in [link].

Operation	Words	Example	Expression

Operation	Words	Example	Expression
Addition	plus sum increased by more than total of added to	1 plus 2 the sum of 3 and 4 5 increased by 6 8 more than 7 the total of 9 and 5 6 added to 4	$1 + 2$ $3 + 4$ $5 + 6$ $7 + 8$ $9 + 5$ $4 + 6$

Example:
Exercise:

Problem: Translate and simplify: the sum of 19 and 23.

Solution:
Solution

The word *sum* tells us to add. The words *of* 19 *and* 23 tell us the addends.

	The sum of 19 and 23
Translate.	$19 + 23$
Add.	42
	The sum of 19 and 23 is 42.

Note:
Exercise:

Problem: Translate and simplify: the sum of 17 and 26.

Solution:

Translate: $17 + 26$; Simplify: 43

Note:
Exercise:

Problem: Translate and simplify: the sum of 28 and 14.

Solution:

Translate: $28 + 14$; Simplify: 42

Example:
Exercise:

Problem: Translate and simplify: 28 increased by 31.

Solution:
Solution

The words *increased by* tell us to add. The numbers given are the addends.

	28 increased by 31.
Translate.	$28 + 31$
Add.	59

	So 28 increased by 31 is 59.

Note:
Exercise:

Problem: Translate and simplify: 29 increased by 76.

Solution:

Translate: $29 + 76$; Simplify 105

Note:
Exercise:

Problem: Translate and simplify: 37 increased by 69.

Solution:

Translate 37 + 69; Simplify 106

Add Whole Numbers in Applications

Plan for Solving Real-World Problems

- Read the problem to determine what we are looking for.
- Write a word phrase that gives the information to find it.
- Translate the word phrase into math notation.
- Simplify.
- Write a sentence to answer the question.

Example:
Exercise:

Problem:

Hao earned grades of $87, 91, 83, 82,$ and 89 on the five tests of the semester. What is the total number of points he earned on the five tests?

Solution:
Solution

We are asked to find the total number of points on the tests.

Write a phrase.	the sum of points on the tests
Translate to math notation.	$87 + 91 + 83 + 82 + 89$
Then we simplify by adding.	
Since there are several numbers, we will write them vertically.	$\begin{array}{r} \overset{2}{8}7 \\ 91 \\ 83 \\ 82 \\ \underline{+89} \\ 432 \end{array}$
Write a sentence to answer the question.	Hao earned a total of 432 points.

Notice that we added *points*, so the sum is 432 *points*. It is important to include the appropriate units in all answers to applications problems.

The Associative Property of Addition

Addition is so familiar to us that we may use properties that addition has without even realizing it.
In a previous problem we just calculated 87 + 91 + 83 + 82 + 89. Since there aren't any we parenthesis, we should add from left to right. That means we should have done 87 + 91 = 178 first. Then we should have taken that sum and added the next number: 178 + 83 = 261, and continued that way until finished eventually getting 432. But that probably isn't the way you did the problem. You probably added all of the ones place first and regrouped (carried) into the tens place and then added that column. Why do we get the same answer either way?

When we add column by column, even for just two numbers, we are actually breaking those numbers apart using expanded form, rearranging the addends, and then finding the sum.
Consider 23 + 45 = 68. Rewrite using the expanded form.
23 + 45 = (20 + 3) + (40 + 5). Rearrange.
(3 + 5) + (20 + 40). And then do the addition.
8 + 60 Add again.
68.
Notice that this closely matches the actual calculation you would do for 23 + 45. The 3 and 5 are added first, then the 2 and 4 are added in the tens place. We don't normally think of all the steps involved.

The Associative Property of Addition says that the order we do additions does not matter for the final result.

Associative Property of Addition
$(a + b) + c = a + (b + c)$

For example, (2 + 3) + 4 = 2 + (3 + 4).
When we add on the left side we get 5 + 4 = 9, and when we add on the right side we get 2 + 7 = 9.
Either gets the same final result even though the intermediate results are different.

The Associative Property of Addition combined with the Commutative Property of Addition allows us to rearrange the order of the addends and how they are grouped. We do that when we add numbers with more than one digit all of the time! Here we are just recognizing and naming properties that allow us to do this.

Note:
Exercise:

Problem:

Mark is training for a bicycle race. Last week he rode 18 miles on Monday, 15 miles on Wednesday, 26 miles on Friday, 49 miles on Saturday, and 32 miles on Sunday. What is the total number of miles he rode last week?

Solution:

He rode 140 miles.

Note:
Exercise:

Problem:

Lincoln Middle School has three grades. The number of students in each grade is $230, 165, \text{and } 325$. What is the total number of students?

Solution:

The total number is 720 students.

Measuring Length

The average length of a newborn human is approximately 20 inches. The average length of a newborn giraffe is approximately 6 feet. Since 20 is greater than 6, does that mean that human newborns are taller than giraffe newborns?

We know the newborn giraffe is much taller, so what is the explanation? The units of measurement, inches and feet, are different so they should not be directly compared. Inches are much smaller than feet; it takes 12 inches to be the same length as 1 foot.

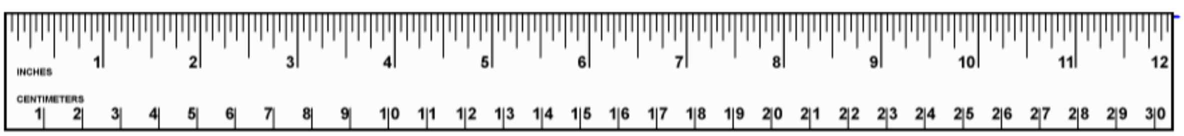
Tools such as rulers, yardsticks, tape measures are used to measure length or distance. We measure length along a path from a starting point to an ending point.

Most often, this is a straight path, but not always. For example, a tailor might use a flexible tape measure to find the distance around your waist or neck.

A ruler is very much like a number line. There is a mark for the starting point, 0, and then one unit to the right is “1”. Moving one unit further to the right is “2”, and this continues until the end of the ruler.

Often rulers have markings on them for two different units of measurement: inches and centimeters. Inches are bigger than centimeters, so for the same length the inches reading will be a smaller number than the centimeter reading. 30 centimeters is a little smaller than 12 inches while 31 centimeters is a little bigger than 12 inches. If a ruler is 12 inches long, the last centimeter marked on the ruler is 30 because 31 will not quite fit.

Note: This ruler is not to scale.

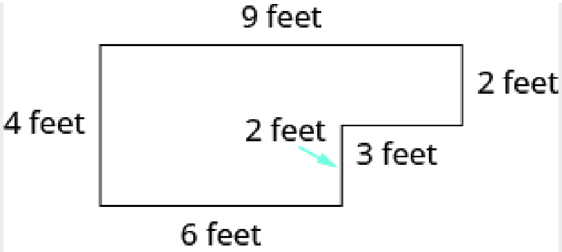


Video
For more information and help with finding the perimeter, please watch this video: https://www.youtube.com/watch?v=AAY1bsazcgM

Some application problems involve shapes. For example, a person might need to know the distance around a garden to put up a fence or around a picture to frame it. The **perimeter** is the distance around a geometric figure. The perimeter of a figure is the sum of the lengths of its sides.

Example:
Exercise:

Problem: Find the perimeter of the patio shown.

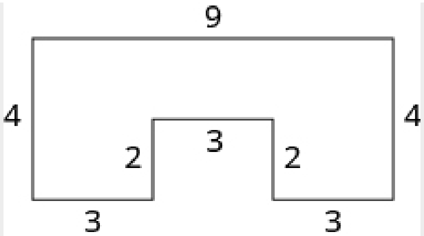


Solution:
Solution

We are asked to find the perimeter.	
Write a phrase.	the sum of the sides
Translate to math notation.	$4+6+2+3+2+9$
Simplify by adding.	26
Write a sentence to answer the question.	
We added feet, so the sum is 26 feet.	The perimeter of the patio is 26 feet.

Note:
Exercise:

Problem: Find the perimeter of each figure. All lengths are in inches.

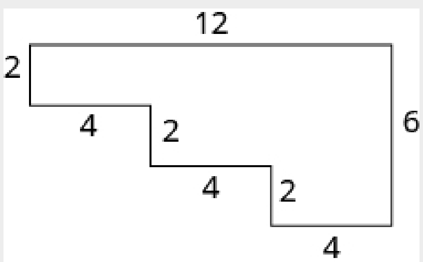


Solution:

The perimeter is 30 inches.

Note:
Exercise:

Problem: Find the perimeter of each figure. All lengths are in inches.



Solution:

The perimeter is 36 inches.

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Adding Two-Digit Numbers with base-10 blocks
- Adding Three-Digit Numbers with base-10 blocks
- Adding Whole Numbers

Key Concepts

- **Addition Notation** To describe addition, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Addition	$+$	$3+4$	three plus four	the sum of 3 and 4

- **Identity Property of Addition**
    - The sum of any number a and 0 is the number. $a+0=a$ $0+a=a$
- **Commutative Property of Addition**
    - Changing the order of the addends a and b does not change their sum. $a+b=b+a$.
- **Associative Property of Addition**
    - Changing the grouping of the addends a, b and c does not change their sum. $(a+b)+c=a+(b+c)$.
- **Add whole numbers.**

  Write the numbers so each place value lines up vertically.
  Add the digits in each place value. Work from right to left starting with the ones place. If a sum in a place value is more than 9, carry to the next place value.
  Continue adding each place value from right to left, adding each place value and carrying if needed.

Exercises

Practice Makes Perfect

Use Addition Notation

In the following exercises, translate the following from math expressions to words.

Exercise:

Problem: $5 + 2$

Solution:

five plus two; the sum of 5 and 2.

Exercise:

Problem: $6 + 3$

Exercise:

Problem: $13 + 18$

Solution:

thirteen plus eighteen; the sum of 13 and 18.

Exercise:

Problem: $15 + 16$

Exercise:

Problem: $214 + 642$

Solution:

two hundred fourteen plus six hundred forty-two; the sum of 214 and 642

Exercise:

Problem: $438 + 113$

Model Addition of Whole Numbers

In the following exercises, model the addition.

Exercise:

Problem: $2 + 4$

Solution:

$2 + 4 = 6$

Exercise:

Problem: $5 + 3$

Exercise:

Problem: $8 + 4$

Solution:

$8 + 4 = 12$

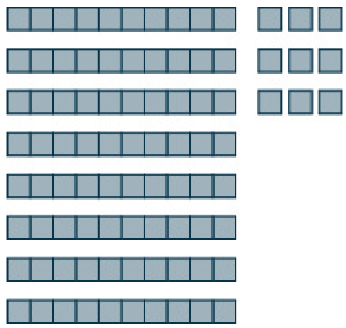
Exercise:

Problem: $5 + 9$

Exercise:

Problem: $14 + 75$

Solution:



$14 + 75 = 89$

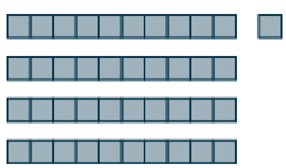
Exercise:

Problem: $15 + 63$

Exercise:

Problem: $16 + 25$

Solution:



$16 + 25 = 41$

Exercise:

Problem: $14 + 27$

Add Whole Numbers

In the following exercises, fill in the missing values in each chart.

Exercise:

Problem:

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2		4	5	6	7		9
1	1	2	3	4			7	8	9	
2		3	4	5	6		8			11
3	3		5		7	8		10		12
4	4	5			8	9		11	12	
5	5	6	7	8			11		13	
6	6	7	8		10			13		15
7			9	10		12			15	16
8	8	9		11			14		16	
9	9	10	11		13	14			17	

Solution:

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Exercise:

Problem:

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4		6		8	9
1	1	2	3		5	6		8		10
2	2		4		6	7		9	10	
3		4		6			9		11	
4	4	5	6	7			10	11		13
5	5	6		8	9		11	12	13	
6			8	9			12	13		15
7	7	8		10		12			15	16
8	8	9	10		12		14		16	17
9			11	12	13			16		

Exercise:

Problem:

+	3	4	5	6	7	8	9
6							
7							
8							
9							

Solution:

+	3	4	5	6	7	8	9
6	9	10	11	12	13	14	15
7	10	11	12	13	14	15	16
8	11	12	13	14	15	16	17
9	12	13	14	15	16	17	18

Exercise:

Problem:

+	6	7	8	9
3				
4				
5				
6				
7				
8				
9				

Exercise:

Problem:

+	5	6	7	8	9
5					
6					
7					
8					
9					

Solution:

+	5	6	7	8	9
5	10	11	12	13	14
6	11	12	13	14	15
7	12	13	14	15	16
8	13	14	15	16	17
9	14	15	16	17	18

Exercise:

Problem:

+	6	7	8	9
6				
7				
8				
9				

In the following exercises, add.

Exercise:

Problem:

ⓐ $0+13$
ⓑ $13+0$

Solution:

ⓐ 13
ⓑ 13

Exercise:

Problem:

ⓐ $0 + 5{,}280$
ⓑ $5{,}280 + 0$

Exercise:

Problem:

ⓐ $8 + 3$
ⓑ $3 + 8$

Solution:

ⓐ 11
ⓑ 11

Exercise:

Problem:

ⓐ $7 + 5$
ⓑ $5 + 7$

Exercise:

Problem: $45 + 33$

Solution:

78

Exercise:

Problem: $37 + 22$

Exercise:

Problem: $71 + 28$

Solution:

99

Exercise:

Problem: $43 + 53$

Exercise:

Problem: $26 + 59$

Solution:

85

Exercise:

Problem: $38 + 17$

Exercise:

Problem: $64 + 78$

Solution:

142

Exercise:

Problem: $92 + 39$

Exercise:

Problem: $168 + 325$

Solution:

493

Exercise:

Problem: $247 + 149$

Exercise:

Problem: $584 + 277$

Solution:

861

Exercise:

Problem: $175 + 648$

Exercise:

Problem: $832 + 199$

Solution:

1,031

Exercise:

Problem: $775 + 369$

Exercise:

Problem: $6{,}358 + 492$

Solution:

6,850

Exercise:

Problem: $9{,}184 + 578$

Exercise:

Problem: $3{,}740 + 18{,}593$

Solution:

22,333

Exercise:

Problem: $6{,}118 + 15{,}990$

Exercise:

Problem: $485{,}012 + 619{,}848$

Solution:

$1{,}104{,}860$

Exercise:

Problem: $368{,}911 + 857{,}289$

Exercise:

Problem: $24{,}731 + 592 + 3{,}868$

Solution:

$29{,}191$

Exercise:

Problem: $28{,}925 + 817 + 4{,}593$

Exercise:

Problem: $8{,}015 + 76{,}946 + 16{,}570$

Solution:

$101{,}531$

Exercise:

Problem: $6{,}291 + 54{,}107 + 28{,}635$

Translate Word Phrases to Math Notation

In the following exercises, translate each phrase into math notation and then simplify.

Exercise:

Problem: the sum of 13 and 18

Solution:

$13 + 18 = 31$

Exercise:

Problem: the sum of 12 and 19

Exercise:

Problem: the sum of 90 and 65

Solution:

$90 + 65 = 155$

Exercise:

Problem: the sum of 70 and 38

Exercise:

Problem: 33 increased by 49

Solution:

$33 + 49 = 82$

Exercise:

Problem: 68 increased by 25

Exercise:

Problem: 250 more than 599

Solution:

$250 + 599 = 849$

Exercise:

Problem: 115 more than 286

Exercise:

Problem: the total of 628 and 77

Solution:

$628 + 77 = 705$

Exercise:

Problem: the total of 593 and 79

Exercise:

Problem: $1{,}482$ added to 915

Solution:

$915 + 1{,}482 = 2{,}397$

Exercise:

Problem: $2{,}719$ added to 682

Add Whole Numbers in Applications

In the following exercises, solve the problem.

Exercise:

Problem:

Home remodeling Sophia remodeled her kitchen and bought a new range, microwave, and dishwasher. The range cost $1,100, the microwave cost $250, and the dishwasher cost $525. What was the total cost of these three appliances?

Solution:

The total cost was $1,875.

Exercise:

Problem:

Sports equipment Aiden bought a baseball bat, helmet, and glove. The bat cost $299, the helmet cost $35, and the glove cost $68. What was the total cost of Aiden's sports equipment?

Exercise:

Problem:

Bike riding Ethan rode his bike 14 miles on Monday, 19 miles on Tuesday, 12 miles on Wednesday, 25 miles on Friday, and 68 miles on Saturday. What was the total number of miles Ethan rode?

Solution:

Ethan rode 138 miles.

Exercise:

Problem:

Business Chloe has a flower shop. Last week she made 19 floral arrangements on Monday, 12 on Tuesday, 23 on Wednesday, 29 on Thursday, and 44 on Friday. What was the total number of floral arrangements Chloe made?

Exercise:

Problem:

Apartment size Jackson lives in a 7 room apartment. The number of square feet in each room is 238, 120, 156, 196, 100, 132, and 225. What is the total number of square feet in all 7 rooms?

Solution:

The total square footage in the rooms is 1,167 square feet.

Exercise:

Problem:

Weight Seven men rented a fishing boat. The weights of the men were $175, 192, 148, 169, 205, 181,$ and 225 pounds. What was the total weight of the seven men?

Exercise:

Problem:

Salary Last year Natalie's salary was $\$82{,}572$. Two years ago, her salary was $\$79{,}316$, and three years ago it was $\$75{,}298$. What is the total amount of Natalie's salary for the past three years?

Solution:

Natalie's total salary is $237,186.

Exercise:

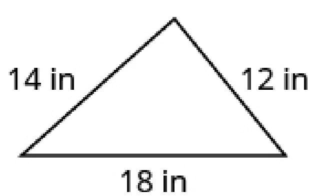
Problem:

Home sales Emma is a realtor. Last month, she sold three houses. The selling prices of the houses were $\$292{,}540$, $\$505{,}875$, and $\$423{,}699$. What was the total of the three selling prices?

In the following exercises, find the perimeter of each figure.

Exercise:

Problem:

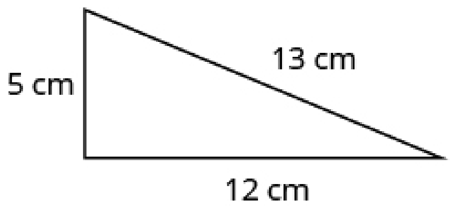


Solution:

The perimeter of the figure is 44 inches.

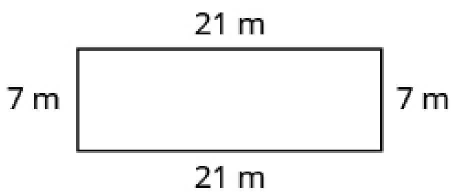
Exercise:

Problem:



Exercise:

Problem:

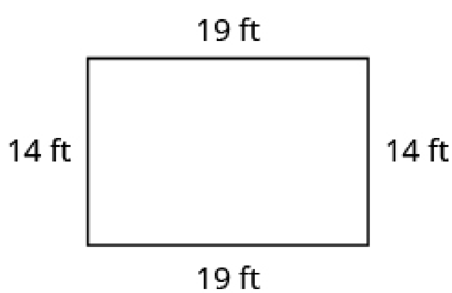


Solution:

The perimeter of the figure is 56 meters.

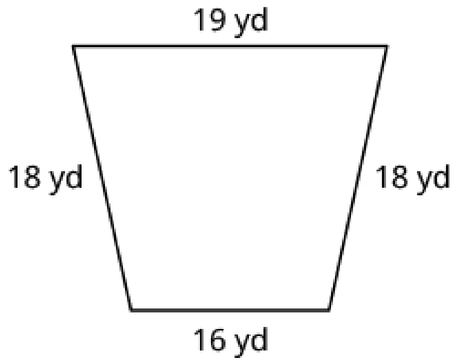
Exercise:

Problem:



Exercise:

Problem:

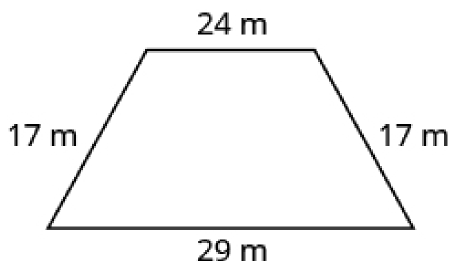


Solution:

The perimeter of the figure is 71 yards.

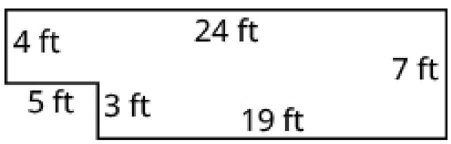
Exercise:

Problem:



Exercise:

Problem:

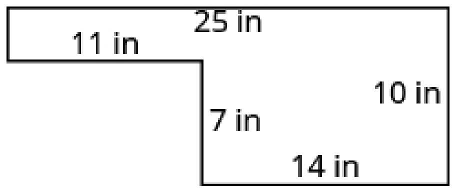


Solution:

The perimeter of the figure is 62 feet.

Exercise:

Problem:



Everyday Math

Exercise:

Problem:

Calories Paulette had a grilled chicken salad, ranch dressing, and a 16-ounce drink for lunch. On the restaurant’s nutrition chart, she saw that each item had the following number of calories:

Grilled chicken salad – 320 calories
Ranch dressing – 170 calories
16-ounce drink – 150 calories

What was the total number of calories of Paulette’s lunch?

Solution:

The total number of calories was 640.

Exercise:

Problem:

Calories Fred had a grilled chicken sandwich, a small order of fries, and a 12-oz chocolate shake for dinner. The restaurant’s nutrition chart lists the following calories for each item:

Grilled chicken sandwich – 420 calories
Small fries – 230 calories
12-oz chocolate shake – 580 calories

What was the total number of calories of Fred’s dinner?

Exercise:

Problem:

Test scores A students needs a total of 400 points on five tests to pass a course. The student scored $82, 91, 75, 88,$ and 70. Did the student pass the course?

Solution:

Yes, he scored 406 points.

Exercise:

Problem:

Elevators The maximum weight capacity of an elevator is 1150 pounds. Six men are in the elevator. Their weights are $210, 145, 183, 230, 159,$ and 164 pounds. Is the total weight below the elevators’ maximum capacity?

Writing Exercises

Exercise:

Problem:

How confident do you feel about your knowledge of the addition facts? If you are not fully confident, what will you do to improve your skills?

Solution:

Answers will vary.

Exercise:

Problem: How have you used models to help you learn the addition facts?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don’t get it!
use addition notation.			
model addition of whole numbers.			
add whole numbers without models.			
explain when and how addition can be more efficient than counting.			
recognize and explain the commutative and associative properties of addition.			
translate word phrases to math notation.			
add whole numbers in applications.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

addend
: A number that is added.

expression
: A math statement that includes numbers and operations.

perimeter
: The perimeter is the distance around a geometric figure.

sum
: The sum is the result of adding two or more numbers.

variable
: A letter or symbol used to stand for potentially any value.

Subtract Whole Numbers
By the end of this section, you will be able to:

- Use subtraction notation
- Model subtraction of whole numbers
- Subtract whole numbers
- Identify properties of subtraction
- Translate word phrases to math notation
- Subtract whole numbers in applications

Note:
Before you get started, take this readiness quiz.

1. Model $3+4$ using base-ten blocks.
   If you missed this problem, review [link].
2. Add: $324+586$.
   If you missed this problem, review [link].

Use Subtraction Notation

Suppose there are seven bananas in a bowl. Elana uses three of them to make a smoothie. How many bananas are left in the bowl? To answer the question, we subtract three from seven. When we subtract, we take one number away from another to find the **difference**. The notation we use to subtract 3 from 7 is
Equation:

$$7-3$$

We read $7-3$ as *seven minus three* and the result is *the difference of seven and three*.

Note:
Subtraction Notation
To describe subtraction, we can use symbols and words.

Operation	**Notation**	**Expression**	**Read as**	**Result**
Subtraction	$-$	$7-3$	seven minus three	the difference of 7 and 3

Example:
Exercise:

Problem:

Translate from math notation to words: ⓐ $8-1$ ⓑ $26-14$.

Solution:
Solution

- ⓐ We read this as *eight minus one*. The result is *the difference of eight and one*.
- ⓑ We read this as *twenty-six minus fourteen*. The resuilt is *the difference of twenty-six and fourteen*.

Note:
Translate from math notation to words:
Exercise:

Problem:

ⓐ $12 - 4$
ⓑ $29 - 11$

Solution:

ⓐ twelve minus four; the difference of twelve and four
ⓑ twenty-nine minus eleven; the difference of twenty-nine and eleven

Note:
Translate from math notation to words:
Exercise:

Problem:

ⓐ $11 - 2$
ⓑ $29 - 12$

Solution:

ⓐ eleven minus two; the difference of eleven and two
ⓑ twenty-nine minus twelve; the difference of twenty-nine and twelve

Model Subtraction of Whole Numbers

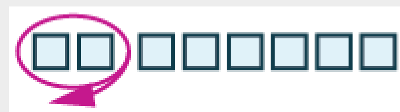
A model can help us visualize the process of subtraction much as it did with addition. Again, we will use base-10 blocks. Remember a block represents 1 and a rod represents 10. Let’s start by modeling the subtraction expression we just considered, $7 - 3$.

We start by modeling the first number, 7.	
Now take away the second number, 3. We'll circle 3 blocks to show that we are taking them away.	
Count the number of blocks remaining.	
There are 4 ones blocks left.	We have shown that $7 - 3 = 4$.

Example:
Exercise:

Problem: Model the subtraction: $8 - 2$.

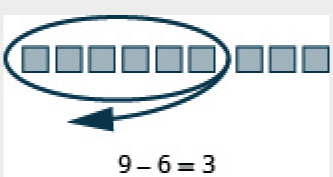
Solution:
Solution

$8-2$ means the difference of 8 and 2.	
Model the first, 8.	
Take away the second number, 2.	
Count the number of blocks remaining.	
There are 6 ones blocks left.	We have shown that $8-2=6$.

Note:
Exercise:

Problem: Model: $9-6$.

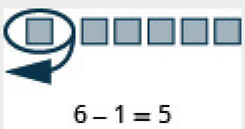
Solution:



Note:
Exercise:

Problem: Model: $6 - 1$.

Solution:

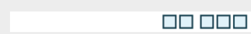


Example:
Exercise:

Problem: Model the subtraction: $13 - 8$.

Solution:
Solution

Model the first number, 13. We use 1 ten and 3 ones.	
Take away the second number, 8. However, there are not 8 ones, so we will exchange	

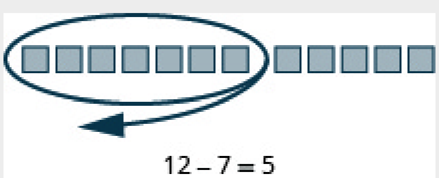
the 1 ten for 10 ones.	
Now we can take away 8 ones.	
Count the blocks remaining.	
There are five ones left.	We have shown that $13 - 8 = 5$.

As we did with addition, we can describe the models as ones blocks and tens rods, or we can simply say ones and tens.

Note:
Exercise:

Problem: Model the subtraction: $12 - 7$.

Solution:

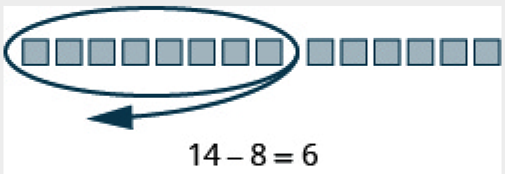


Note:

Exercise:

Problem: Model the subtraction: 14 – 8.

Solution:

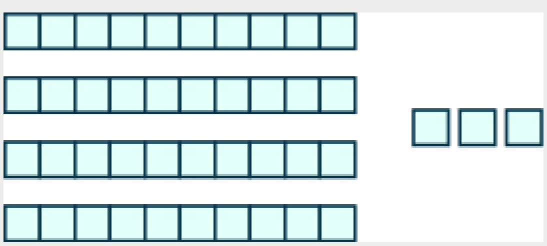


Example:
Exercise:

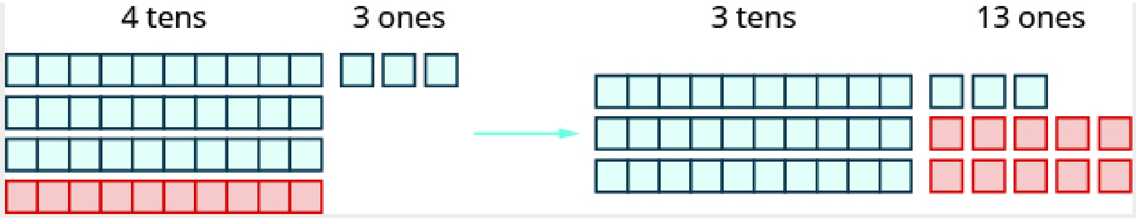
Problem: Model the subtraction: 43 – 26.

Solution:
Solution

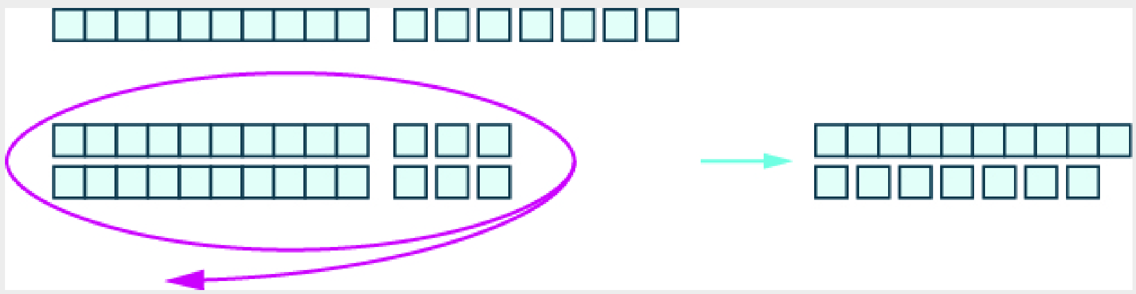
Because 43 – 26 means 43 take away 26, we begin by modeling the 43.



Now, we need to take away 26, which is 2 tens and 6 ones. We cannot take away 6 ones from 3 ones. So, we exchange 1 ten for 10 ones.



Now we can take away 2 tens and 6 ones.



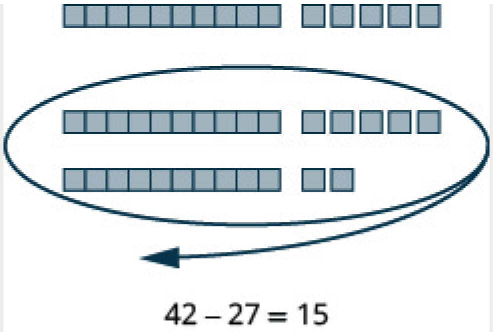
Count the number of blocks remaining. There is 1 ten and 7 ones, which is 17.

$43 - 26 = 17$

Note:
Exercise:

Problem: Model the subtraction: $42 - 27$.

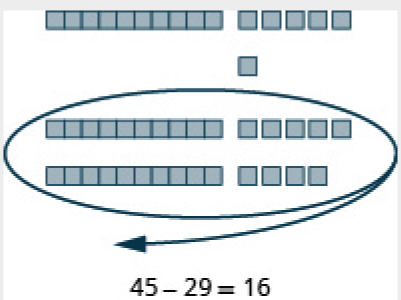
Solution:



Note:
Exercise:

Problem: Model the subtraction: $45 - 29$.

Solution:



Properties of Subtraction

Just as addition has some interesting properties (identity, commutative, and associative) subtraction has interesting properties too. Since addition and subtraction are related, we might expect there to be similar properties for subtraction, but we need to investigate to be sure.

Identity Property of Subtraction
The identity property for addition is: $a + 0 = a$. A similar property for subtraction is: $a - 0 = a$. Examples are: $5 - 0 = 5$ and $17 - 0 = 17$.

No matter how many examples we give it does not prove the property is true for any value of a. In order for us to accept this as a property it must be true for every value of a. That requires more general reasoning. If I have an amount a and remove nothing from it then my amount is still a.

Here is another property related to the identity property of subtraction: $a - a = 0$. In words this says, any number minus the same number is zero. Examples are $5 - 5 = 0$ and $17 - 17 = 0$. Just giving examples does not prove that this is true in general. In your own words, explain why this is true.

Commutative Property of Subtraction?
The commutative property of addition says that $a + b = b + a$. The order of the addends does not matter. What about for subtraction. Does $a - b = b - a$?

Sometimes it helps to put numbers in for the variables to see if a property works. If it works for those numbers the property might be true but that doesn't prove that it is true with other values. But if it is false then we know that our property isn't a real property. It takes just one counterexample to show the property is false.

Let $a = 7$ and $b = 3$. Then $a - b$ is $7 - 3 = 4$, while $b - a$ is $3 - 7$ which we can not compute with whole numbers because one can not take a larger amount away from a smaller amount. Therefore there is no commutative property of subtraction.

Associative Property of Subtraction?
The associative property of addition says that $(a + b) + c = a + (b + c)$.
If there is an associative property of subtraction it would look the same except every plus sign would be replaced with a minus sign: $(a - b) - c = a - (b - c)$.

This also does not work. Try $a = 5$, $b = 3$, and $c = 1$ to show that it is not true.

(5 - 3) - 1 = 2 - 1 = 1 while 5 - (3 - 1) = 5 - 2 = 3. Therefore there is no associative property of subtraction.

Subtract Whole Numbers

Addition and subtraction are inverse operations. Addition undoes subtraction, and subtraction undoes addition.

If you have $7 and loan $3 then you have $4 left. If you then get the $3 paid back you have $7 again.
7 - 3 + 3 = 4 + 3 = 7
Subtracting 3 was undone by adding 3.

It works the other way too.
If you have $4 and find $3 and then lose $3 you'll be back where you started with $4.
4 + 3 - 3 = 7 - 3 = 4.
Adding 3 was undone by subtracting 3.

One way to know that $7 - 3 = 4$ is because $4 + 3 = 7$. Knowing all the addition number facts will help with subtraction. We can check subtraction by adding.
Equation:

$$\begin{array}{rcr} 7-3=4 & \text{because} & 4+3=7 \\ 13-8=5 & \text{because} & 5+8=13 \\ 43-26=17 & \text{because} & 17+26=43 \end{array}$$

Example:
Exercise:

Problem: Subtract and then check by adding:

ⓐ $9 - 7$

ⓑ $8 - 3$.

Solution:
Solution

ⓐ	
	$9 - 7$
Subtract 7 from 9.	2
Check with addition. $2 + 7 = 9\checkmark$	

ⓑ	
	$8 - 3$
Subtract 3 from 8.	5
Check with addition. $5 + 3 = 8\checkmark$	

Note:
Exercise:

Problem: Subtract and then check by adding:

$7 - 0$

Solution:

$7 - 0 = 7; 7 + 0 = 7$

Note:
Exercise:

Problem: Subtract and then check by adding:

$6 - 2$

Solution:

$6 - 2 = 4; 2 + 4 = 6$

To subtract numbers with more than one digit, it is usually easier to write the numbers vertically in columns just as we did for addition. Align the digits by place value, and then subtract each column starting with the ones and then working to the left.

Example:
Exercise:

Problem: Subtract and then check by adding: $89 - 61$.

Solution:
Solution

Write the numbers so the ones and tens digits line up vertically.	$\begin{array}{r} 89 \\ -61 \\ \hline \end{array}$
Subtract the digits in each place value. Subtract the ones: $9 - 1 = 8$ Subtract the tens: $8 - 6 = 2$	$\begin{array}{r} 89 \\ -61 \\ \hline 28 \end{array}$
Check using addition. $\begin{array}{r} 28 \\ +61 \\ \hline 89 \end{array}$	

Our answer is correct.

Note:
Exercise:

Problem: Subtract and then check by adding: $86 - 54$.

Solution:

86 − 54 = 32 because 54 + 32 = 86

Note:
Exercise:

Problem: Subtract and then check by adding: $99 - 74$.

Solution:

99 − 74 = 25 because 74 + 25 = 99

When we modeled subtracting 26 from 43, we exchanged 1 ten for 10 ones. When we do this without the model, we say we borrow 1 from the tens place and add 10 to the ones place.

Note:
Find the difference of whole numbers.

Write the numbers so each place value lines up vertically.
Subtract the digits in each place value. Work from right to left starting with the ones place. If the digit on top is less than the digit below, borrow as needed.
Continue subtracting each place value from right to left, borrowing if needed.
Check by adding.

Why do we start with the digits on the right instead of the digits on the left?
If we started on the left and wrote down the answer we would have to go back and correct it if we needed to borrow when we got further to the right.

Example:
Exercise:

Problem: Subtract: $43 - 26$.

Solution:
Solution

Write the numbers so each place value lines up vertically.	
Subtract the ones. We cannot subtract 6 from 3, so we borrow 1 ten. This makes 3 tens and 13 ones. We write these numbers above each place and cross out the original digits.	
Now we can subtract the ones. $13 - 6 = 7$. We write the 7 in the ones place in the difference.	
Now we subtract the tens. $3 - 2 = 1$. We write the 1 in the tens place in the difference.	
Check by adding. $\begin{array}{r} 17 \\ +26 \\ \hline 43 \checkmark \end{array}$ Our answer is correct.	

Note:
Exercise:

Problem: Subtract and then check by adding: $93 - 58$.

Solution:

$93 - 58 = 35$ because $58 + 35 = 93$

Note:
Exercise:

Problem: Subtract and then check by adding: $81 - 39$.

Solution:

$81 - 39 = 42$ because $42 + 39 = 81$

Example:
Exercise:

Problem: Subtract and then check by adding: $207 - 64$.

Solution:
Solution

Write the numbers so each place value lines up vertically.	
Subtract the ones. $7 - 4 = 3$. Write the 3 in the ones place in the difference. Write the 3 in the ones place in the difference.	
Subtract the tens. We cannot subtract 6 from 0 so we borrow 1 hundred and add 10 tens to the 0 tens we had. This makes a total of 10 tens. We write 10 above the tens place and cross out the 0. Then we cross out the 2 in the hundreds place and write 1 above it.	
Now we subtract the tens. $10 - 6 = 4$. We write the 4 in the tens place in the difference.	
Finally, subtract the hundreds. There is no digit in the hundreds place in the bottom number so we can imagine a 0 in that place. Since $1 - 0 = 1$, we write 1 in the hundreds place in the difference.	
Check by adding. $\begin{array}{r} {}^{1} \\ 143 \\ +\ 64 \\ \hline 207\checkmark \end{array}$ Our answer is correct.	

Note:
Exercise:

Problem: Subtract and then check by adding: $439 - 52$.

Solution:

439 − 52 = 387 because 387 + 52 = 439

Note:
Exercise:

Problem: Subtract and then check by adding: $318 - 75$.

Solution:

318 − 75 = 243 because 243 + 75 = 318

Example:
Exercise:

Problem: Subtract and then check by adding: $910 - 586$.

Solution:
Solution

Write the numbers so each place value lines up vertically.	

Subtract the ones. We cannot subtract 6 from 0, so we borrow 1 ten and add 10 ones to the 10 ones we had. This makes 10 ones. We write a 0 above the tens place and cross out the 1. We write the 10 above the ones place and cross out the 0. Now we can subtract the ones. $10 - 6 = 4$.

Write the 4 in the ones place of the difference.

Subtract the tens. We cannot subtract 8 from 0, so we borrow 1 hundred and add 10 tens to the 0 tens we had, which gives us 10 tens. Write 8 above the hundreds place and cross out the 9. Write 10 above the tens place.

Now we can subtract the tens. $10 - 8 = 2$.

Subtract the hundreds place. $8 - 5 = 3$ Write the 3 in the hundreds place in the difference.

Check by adding.

$$\begin{array}{r} \overset{1}{3}\,\overset{1}{2}\,4 \\ +\ \ 5\,8\,6 \\ \hline 9\,1\,0\ \checkmark \end{array}$$

Our answer is correct.

Note:
Exercise:

Problem: Subtract and then check by adding: $832 - 376$.

Solution:

832 − 376 = 456 because 456 + 376 = 832

Note:
Exercise:

Problem: Subtract and then check by adding: $847 - 578$.

Solution:

847 − 578 = 269 because 269 + 578 = 847

Example:
Exercise:

Problem: Subtract and then check by adding: $2{,}162 - 479$.

Solution:
Solution

Write the numbers so each place values line up vertically.	$\begin{array}{r} 2,162 \\ -\ 479 \\ \hline \end{array}$

Subtract the ones. Since we cannot subtract 9 from 2, borrow 1 ten and add 10 ones to the 2 ones to make 12 ones. Write 5 above the tens place and cross out the 6. Write 12 above the ones place and cross out the 2.	$\begin{array}{rrrr} & & 5 & 12 \\ 2, & 1 & \cancel{6} & \cancel{2} \\ - & 4 & 7 & 9 \\ \hline \end{array}$
Now we can subtract the ones.	$12-9=3$
Write 3 in the ones place in the difference.	$\begin{array}{rrrr} & & 5 & 12 \\ 2, & 1 & \cancel{6} & \cancel{2} \\ - & 4 & 7 & 9 \\ \hline & & & 3 \end{array}$
Subtract the tens. Since we cannot subtract 7 from 5, borrow 1 hundred and add 10 tens to the 5 tens to make 15 tens. Write 0 above the hundreds place and cross out the 1. Write 15 above the tens place.	$\begin{array}{rrrr} & & 15 & \\ & 0 & \cancel{5} & 12 \\ 2, & \cancel{1} & \cancel{6} & \cancel{2} \\ - & 4 & 7 & 9 \\ \hline & & & 3 \end{array}$
Now we can subtract the tens.	$15-7=8$
Write 8 in the tens place in the difference.	$\begin{array}{rrrr} & 0 & 15 & 12 \\ 2, & \cancel{1} & \cancel{6} & \cancel{2} \\ - & 4 & 7 & 9 \\ \hline & & 8 & 3 \end{array}$
Now we can subtract the hundreds.	$\begin{array}{rrrr} & 10 & & \\ 1 & \cancel{0} & 15 & 12 \\ \cancel{2}, & \cancel{1} & \cancel{6} & \cancel{2} \\ - & 4 & 7 & 9 \\ \hline & & 8 & 3 \end{array}$

Write 6 in the hundreds place in the difference.	$\begin{array}{r} {\scriptstyle 1\ \ 10\ 15\ 12} \\ \cancel{2},\cancel{1}\cancel{6}\cancel{2} \\ -\ 4\ 7\ 9 \\ \hline 6\ 8\ 3 \end{array}$
Subtract the thousands. There is no digit in the thousands place of the bottom number, so we imagine a 0. $1-0=1$. Write 1 in the thousands place of the difference.	$\begin{array}{r} {\scriptstyle 1\ \ 10\ \cancel{15}\ \cancel{12}} \\ \cancel{2},\cancel{1}\cancel{6}\cancel{2} \\ -\ 4\ 7\ 9 \\ \hline 1,6\ 8\ 3 \end{array}$
Check by adding. $\begin{array}{r} {\scriptstyle 1\ \ \ 1\ 1\ } \\ 1,683 \\ +\ \ 479 \\ \hline 2,162 \checkmark \end{array}$	

Our answer is correct.

Note:
Exercise:

Problem: Subtract and then check by adding: $4,585-697$.

Solution:

4,585 − 697 = 3,888 because 3,888 + 697 = 4,585

Note:
Exercise:

Problem: Subtract and then check by adding: $5,637 - 899$.

Solution:

5,637 − 899 = 4,738 because 4,738 + 899 = 5,637

Translate Word Phrases to Math Notation

As with addition, word phrases can tell us to operate on two numbers using subtraction. To translate from a word phrase to math notation, we look for key words that indicate subtraction. Some of the words that indicate subtraction are listed in [link].

Operation	Word Phrase	Example	Expression
Subtraction	minus	5 minus 1	$5 - 1$
	difference	the difference of 9 and 4	$9 - 4$
	decreased by	7 decreased by 3	$7 - 3$
	less than	5 less than 8	$8 - 5$

Operation	Word Phrase	Example	Expression
	subtracted from	1 subtracted from 6	$6 - 1$

Example:
Exercise:

Problem: Translate and then simplify:

ⓐ the difference of 13 and 8
ⓑ subtract 24 from 43

Solution:
Solution

- ⓐ

  The word *difference* tells us to subtract the two numbers. The numbers stay in the same order as in the phrase.

	the difference of 13 and 8
Translate.	$13 - 8$
Simplify.	5

- ⓑ

  The words *subtract from* tells us to take the second number away from the first. We must be careful to get the order correct.

	subtract 24 from 43
Translate.	$43-24$
Simplify.	19

Note:
Exercise:

Problem: Translate and simplify:

ⓐ the difference of 14 and 9
ⓑ subtract 21 from 37

Solution:

ⓐ 14 − 9 = 5
ⓑ 37 − 21 = 16

Note:

Exercise:

Problem: Translate and simplify:

ⓐ 11 decreased by 6
ⓑ 18 less than 67

Solution:

ⓐ $11 - 6 = 5$
ⓑ $67 - 18 = 49$

Subtract Whole Numbers in Applications

To solve applications with subtraction, we will use the same plan that we used with addition.

- Determine what we are asked to find.
- Write a phrase that gives the information to find it.
- Translate the phrase into math notation.
- Simplify to get the answer.
- Write a sentence to answer the question, using the appropriate units.

Example:
Exercise:

Problem:

The temperature in Chicago one morning was 73 degrees Fahrenheit. A cold front arrived and by noon the temperature was 27 degrees Fahrenheit. What was the difference between the temperature in the morning and the temperature at noon?

Solution:
Solution

We are asked to find the difference between the morning temperature and the noon temperature.

Write a phrase.	the difference of 73 and 27
Translate to math notation. *Difference* tells us to subtract.	$73 - 27$
Then we do the subtraction.	$\begin{array}{r} \overset{6}{\cancel{7}}\,\overset{13}{\cancel{3}} \\ -\ 2\,7 \\ \hline 4\,6 \end{array}$
Write a sentence to answer the question.	The difference in temperatures was 46 degrees Fahrenheit.

Note:
Exercise:

Problem:

The high temperature on June 1^{st} in Boston was 77 degrees Fahrenheit, and the low temperature was 58 degrees Fahrenheit. What was the difference between the high and low temperatures?

Solution:

The difference is 19 degrees Fahrenheit.

Note:
Exercise:

Problem:

The weather forecast for June 2 in St Louis predicts a high temperature of 90 degrees Fahrenheit and a low of 73 degrees Fahrenheit. What is the difference between the predicted high and low temperatures?

Solution:

The difference is 17 degrees Fahrenheit.

Example:
Exercise:

Problem:

A washing machine is on sale for \$399. Its regular price is \$588. What is the difference between the regular price and the sale price?

Solution:
Solution

We are asked to find the difference between the regular price and the sale price.

Write a phrase.	the difference between 588 and 399
Translate to math notation.	$588 - 399$
Subtract.	$\begin{array}{r} \overset{4}{\cancel{5}}\overset{17}{\cancel{8}}\overset{18}{\cancel{8}} \\ -\ 399 \\ \hline 189 \end{array}$
Write a sentence to answer the question.	The difference between the regular price and the sale price is \$189.

Note:
Exercise:

Problem:

A television set is on sale for \$499. Its regular price is \$648. What is the difference between the regular price and the sale price?

Solution:

The difference is \$149.

Note:
Exercise:

Problem:

A patio set is on sale for $149. Its regular price is $285. What is the difference between the regular price and the sale price?

Solution:

The difference is $136.

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Model subtraction of two-digit whole numbers
- Model subtraction of three-digit whole numbers
- Subtract Whole Numbers

Key Concepts

Operation	Notation	Expression	Read as	Result

Operation	Notation	Expression	Read as	Result
Subtraction	$-$	$7-3$	seven minus three	the difference of 7 and 3

- **Subtract whole numbers.**

  Write the numbers so each place value lines up vertically.
  Subtract the digits in each place value. Work from right to left starting with the ones place. If the digit on top is less than the digit below, borrow as needed.
  Continue subtracting each place value from right to left, borrowing if needed.
  Check by adding.

Exercises

Practice Makes Perfect

Use Subtraction Notation

In the following exercises, translate from math notation to words.
Exercise:

Problem: $15-9$

Solution:

fifteen minus nine; the difference of fifteen and nine

Exercise:

Problem: $18-16$

Exercise:

Problem: $42-35$

Solution:

forty-two minus thirty-five; the difference of forty-two and thirty-five

Exercise:

Problem: $83-64$

Exercise:

Problem: $675-350$

Solution:

hundred seventy-five minus three hundred fifty; the difference of six hundred seventy-five and three hundred fifty

Exercise:

Problem: $790-525$

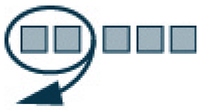
Model Subtraction of Whole Numbers

In the following exercises, model the subtraction.

Exercise:

Problem: $5-2$

Solution:



Exercise:

Problem: $8-4$

Exercise:

Problem: $6-3$

Solution:

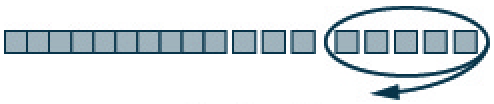
Exercise:

Problem: $7-5$

Exercise:

Problem: $18-5$

Solution:



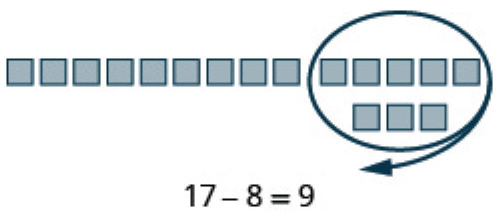
Exercise:

Problem: $19 - 8$

Exercise:

Problem: $17 - 8$

Solution:



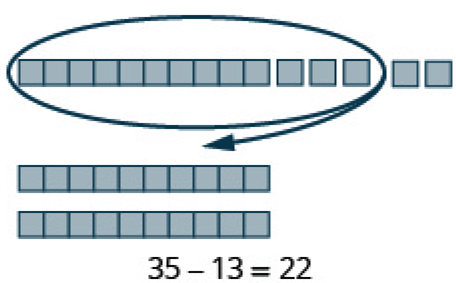
Exercise:

Problem: $17 - 9$

Exercise:

Problem: $35 - 13$

Solution:



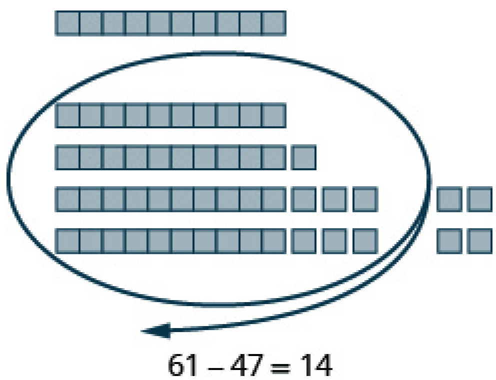
Exercise:

Problem: $32 - 11$

Exercise:

Problem: $61 - 47$

Solution:



Exercise:

Problem: $55 - 36$

Subtract Whole Numbers

In the following exercises, subtract and then check by adding.

Exercise:

Problem: $9 - 4$

Solution:

5

Exercise:

Problem: $9 - 3$

Exercise:

Problem: $8 - 0$

Solution:

8

Exercise:

Problem: $2 - 0$

Exercise:

Problem: $38 - 16$

Solution:

22

Exercise:

Problem: $45 - 21$

Exercise:

Problem: $85 - 52$

Solution:

33

Exercise:

Problem: $99 - 47$

Exercise:

Problem: $493 - 370$

Solution:

123

Exercise:

Problem: $268 - 106$

Exercise:

Problem: $5{,}946 - 4{,}625$

Solution:

$1{,}321$

Exercise:

Problem: $7{,}775 - 3{,}251$

Exercise:

Problem: $75 - 47$

Solution:

28

Exercise:

Problem: $63 - 59$

Exercise:

Problem: $461 - 239$

Solution:

222

Exercise:

Problem: $486 - 257$

Exercise:

Problem: $525 - 179$

Solution:

346

Exercise:

Problem: $542 - 288$

Exercise:

Problem: $6{,}318 - 2{,}799$

Solution:

3,519

Exercise:

Problem: $8{,}153 - 3{,}978$

Exercise:

Problem: $2{,}150 - 964$

Solution:

$1{,}186$

Exercise:

Problem: $4{,}245 - 899$

Exercise:

Problem: $43{,}650 - 8{,}982$

Solution:

$34{,}668$

Exercise:

Problem: $35{,}162 - 7{,}885$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate and simplify.

Exercise:

Problem: The difference of 10 and 3

Solution:

$10 - 3; 7$

Exercise:

Problem: The difference of 12 and 8

Exercise:

Problem: The difference of 15 and 4

Solution:

$15 - 4; 11$

Exercise:

Problem: The difference of 18 and 7

Exercise:

Problem: Subtract 6 from 9

Solution:

$9 - 6; 3$

Exercise:

Problem: Subtract 8 from 9

Exercise:

Problem: Subtract 28 from 75

Solution:

$75 - 28; 47$

Exercise:

Problem: Subtract 59 from 81

Exercise:

Problem: 45 decreased by 20

Solution:

$45 - 20; 25$

Exercise:

Problem: 37 decreased by 24

Exercise:

Problem: 92 decreased by 67

Solution:

$92 - 67; 25$

Exercise:

Problem: 75 decreased by 49

Exercise:

Problem: 12 less than 16

Solution:

$16 - 12; 4$

Exercise:

Problem: 15 less than 19

Exercise:

Problem: 38 less than 61

Solution:

$61 - 38; 23$

Exercise:

Problem: 47 less than 62

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: $76 - 47$

Solution:

29

Exercise:

Problem: $91 - 53$

Exercise:

Problem: $256 - 184$

Solution:

72

Exercise:

Problem: $305 - 262$

Exercise:

Problem: $719 + 341$

Solution:

1,060

Exercise:

Problem: $647 + 528$

Exercise:

Problem: $2{,}015-1{,}993$

Solution:

22

Exercise:

Problem: $2{,}020-1{,}984$

In the following exercises, translate and simplify.

Exercise:

Problem: Seventy-five more than thirty-five

Solution:

75 + 35; 110

Exercise:

Problem: Sixty more than ninety-three

Exercise:

Problem: 13 less than 41

Solution:

41 − 13; 28

Exercise:

Problem: 28 less than 36

Exercise:

Problem: The difference of 100 and 76

Solution:

$100 - 76$; 24

Exercise:

Problem: The difference of 1,000 and 945

Subtract Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature The high temperature on June 2 in Las Vegas was 80 degrees and the low temperature was 63 degrees. What was the difference between the high and low temperatures?

Solution:

The difference between the high and low temperature was 17 degrees

Exercise:

Problem:

Temperature The high temperature on June 1 in Phoenix was 97 degrees and the low was 73 degrees. What was the difference between the high and low temperatures?

Exercise:

Problem:

Class size Olivia’s third grade class has 35 children. Last year, her second grade class had 22 children. What is the difference between the number of children in Olivia’s third grade class and her second grade class?

Solution:

The difference between the third grade and second grade was 13 children.

Exercise:

Problem:

Class size There are 82 students in the school band and 46 in the school orchestra. What is the difference between the number of students in the band and the orchestra?

Exercise:

Problem:

Shopping A mountain bike is on sale for \$399. Its regular price is \$650. What is the difference between the regular price and the sale price?

Solution:

The difference between the regular price and sale price is \$251.

Exercise:

Problem:

Shopping A mattress set is on sale for \$755. Its regular price is \$1,600. What is the difference between the regular price and the sale price?

Exercise:

Problem:

Savings John wants to buy a laptop that costs $\$840$. He has $\$685$ in his savings account. How much more does he need to save in order to buy the laptop?

Solution:

John needs to save $155 more.

Exercise:

Problem:

Banking Mason had $\$1,125$ in his checking account. He spent $\$892$. How much money does he have left?

Everyday Math

Exercise:

Problem:

Road trip Noah was driving from Philadelphia to Cincinnati, a distance of 502 miles. He drove 115 miles, stopped for gas, and then drove another 230 miles before lunch. How many more miles did he have to travel?

Solution:

157 miles

Exercise:

Problem:

Test Scores Sara needs 350 points to pass her course. She scored $75, 50, 70,$ and 80 on her first four tests. How many more points does Sara need to pass the course?

Writing Exercises

Exercise:

Problem: Explain how subtraction and addition are related.

Solution:

Answers may vary.

Exercise:

Problem:

How does knowing addition facts help you to subtract numbers?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use subtraction notation.			
model subtraction of whole numbers.			
subtract whole numbers.			
identify properties of subtraction.			
translate word phrases to math notation.			
subtract whole numbers in applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

difference
: The difference is the result of subtracting two or more numbers.

Multiply Whole Numbers Beginning Level
By the end of this section, you will be able to:

- Use multiplication notation
- Model multiplication of whole numbers where one factor is a single digit
- Identify and use properties of multiplication
- Model multiplication as the area of a rectangle
- Use the distributive property of multiplication
- Multiply whole numbers where one factor is a single digit
- Translate word phrases to math notation
- Multiply whole numbers in applications

Note:
Before you get started, take this readiness quiz.

1. Add: $1{,}683 + 479$.
   If you missed this problem, review [link].
2. Subtract: $605 - 321$.
   If you missed this problem, review [link].

Use Multiplication Notation

Suppose you were asked to find out the number of pennies shown in [link].

Would you count the pennies individually? Or would you count the number of pennies in a row and add that number 3 times.
Equation:

$$8 + 8 + 8$$

Multiplication is a way to represent repeated addition. Repeated addition means the same addend occurs over and over again. So instead of adding three 8s, we could write a multiplication expression.
Equation:

$$3 \times 8$$

We call each number in the multiplication a **factor** and the result the **product**. We read 3×8 as *three times eight*, and the result as *the product of three and eight*.

factor x factor = product

There are several symbols that represent multiplication. These include the symbol $\times$ as well as the dot, $\cdot$, and parentheses $(\ )$.

Note:
Operation Symbols for Multiplication
To describe multiplication, we can use symbols and words.

Operation	**Notation**	**Expression**	**Read as**	**Result**
Multiplication	$\times$ $\cdot$ $(\ )$	3×8 $3 \cdot 8$ $3(8)$	three times eight	the product of 3 and 8

Example:
Exercise:

Problem: Translate from math notation to words:

ⓐ 7×6
ⓑ $12 \cdot 14$
ⓒ $6(13)$

Solution:
Solution

- ⓐ We read this as *seven times six* and the result is *the product of seven and six*.
- ⓑ We read this as *twelve times fourteen* and the result is *the product of twelve and fourteen*.
- ⓒ We read this as *six times thirteen* and the result is *the product of six and thirteen*.

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ 8×7
ⓑ $18 \cdot 11$

Solution:

ⓐ eight times seven ; the product of eight and seven
ⓑ eighteen times eleven ; the product of eighteen and eleven

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $(13)(7)$
ⓑ $5(16)$

Solution:

ⓐ thirteen times seven ; the product of thirteen and seven
ⓑ five times sixteen; the product of five and sixteen

Model Multiplication of Whole Numbers with Counters

There is more than one way to model multiplication. Here we will use counters to help us understand the meaning of multiplication. A counter is any object that can be used for counting. We will use round blue circles or disks.

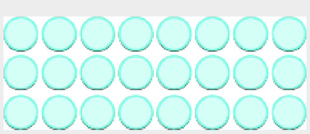
Example:
Exercise:

Problem: Model: 3×8.

Solution:
Solution

To model the product 3×8, we'll start with a row of 8 counters.

The other factor is 3, so we'll make 3 rows of 8 counters.



Count the result, or more efficiently add 8 + 8 + 8. There are 24 counters in all.

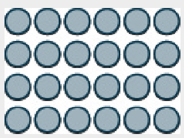
$3 \times 8 = 24$

If you look at the counters sideways, you'll see that we could have also made 8 rows of 3 counters. The product would have been the same. We'll return to this idea later.

Note:
Exercise:

Problem: Model each multiplication: 4×6.

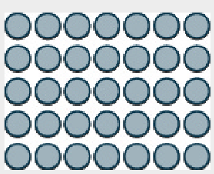
Solution:



Note:
Exercise:

Problem: Model each multiplication: 5×7.

Solution:



Multiply Whole Numbers

In order to multiply without using models, it helps to know all the one digit multiplication facts and how to multiply by 10.

The table below shows the multiplication facts. Each box shows the product of the number down the left column and the number across the top row. If you are unsure about a product, model it. It is important that you either memorize any number fact you do not already know or have a strategy for quickly getting it.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

A multiplication table is a way of organizing multiplication facts so that we can easily see patterns in the facts. Since multiplication can be computed by repetitive addition, that is one way to make the table. For example, the 5s column starts, "5, 10, 15, …" These are the results when we sum 1 five, 2 fives, 3 fives,etc. We can get the next value by adding another 5, and this continues until the end of the chart. We could do this for any column. We could also do this for any row.

Note:
Multiplication Property of Zero
What happens when you multiply a number by zero? That is so easy to remember that it was not put in the chart. Thinking of multiplication as repeated addition, if the addends are zero then no matter how many times it is added, the sum is zero. Also, if the number of addends is zero then the sum is zero. This gives: The product of any number and 0 is 0.
Equation:

$$a \cdot 0 = 0$$
$$0 \cdot a = 0$$

Example:
Exercise:

Problem: Multiply:

ⓐ $0 \cdot 11$
ⓑ $(42)0$

Solution:
Solution

ⓐ	$0 \cdot 11$
The product of any number and zero is zero.	0
ⓑ	$(42)0$
Multiplying by zero results in zero.	0

Note:
Exercise:

Problem: Find each product:

ⓐ $0 \cdot 19$
ⓑ $(39)0$

Solution:

ⓐ 0
ⓑ 0

Note:
Exercise:

Problem: Find each product:

ⓐ $0 \cdot 24$
ⓑ $(57)0$

Solution:

ⓐ 0
ⓑ 0

What happens when you multiply a number by one? Multiplying a number by one does not change the value. For this reason, 1 is called the Multiplicative Identity. An identity for an operation is a number that does not change the value of the other number in the operation. Similarly, 0 is the Additive Identity because adding 0 does not change the value.

Note:
Identity Property of Multiplication
The product of any number and 1 is the number.
Equation:

$$1 \cdot a = a$$
$$a \cdot 1 = a$$

Why is this property true? The two equations have different justifications.

- $1 \cdot a$ interpreted as repeated addition means having one addend, a, nothing is added to it, so it equals a.
- $a \cdot 1$ interpreted as repeated addition means having a addends all equal to 1. Since there are a 1s, as they are summed we get the counting numbers up to and ending with a, so that also equals a.

Example:
Exercise:

Problem: Multiply:

ⓐ $(11)1$
ⓑ $1 \cdot 42$

Solution:
Solution

ⓐ	$(11)1$
The product of any number and one is the number.	11
ⓑ	$1 \cdot 42$
Multiplying by one does not change the value.	42

Note:
Exercise:

Problem: Find each product:

ⓐ $(19)1$
ⓑ $1 \cdot 39$

Solution:

ⓐ 19
ⓑ 39

Note:
Exercise:

Problem: Find each product:

ⓐ$(24)(1)$
ⓑ 1×57

Solution:

ⓐ 24
ⓑ 57

Previously, we learned that the Commutative Property of Addition states that changing the order of addition does not change the sum. We saw that $8+9=17$ gave the same sum as $9+8=17$.

Is this also true for multiplication? Let's look at a few pairs of factors.
Equation:

$$3 \cdot 8 = 24 \qquad 8 \cdot 3 = 24$$

Equation:

$$9 \cdot 7 = 63 \qquad 7 \cdot 9 = 63$$

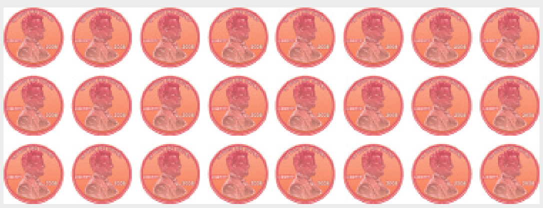
Equation:

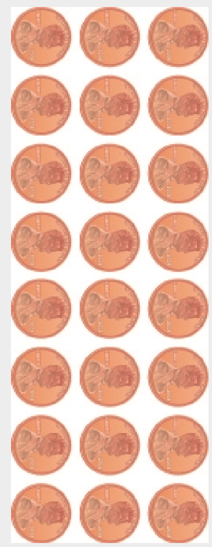
$$8 \cdot 9 = 72 \qquad 9 \cdot 8 = 72$$

Note:

Commutative Property of Multiplication
When the order of the factors is reversed, the product does not change. Recall the picture of 3 x 8 from the beginning of this section. If that is rotated 90 degrees (one quarter turn) in either direction it will look like 8 x 3. Notice that the number of counters remains the same. This is the case for any factors a x b, so a x b must equal b x a.

This property is very good news if you are learning your multiplication facts because you do not have to remember both orders of two factors. For example, 3 x 8 and 8 x 3 are both 24.





Equation:

$$a \cdot b = b \cdot a$$

Example:
Exercise:

Problem: Multiply:

ⓐ $8 \cdot 7$
ⓑ $7 \cdot 8$

Solution:
Solution

ⓐ	$8 \cdot 7$
Multiply.	56
ⓑ	$7 \cdot 8$
Multiply.	56

Changing the order of the factors does not change the product.

Note:
Exercise:

Problem: Multiply:

ⓐ $9 \cdot 6$
ⓑ $6 \cdot 9$

Solution:

54 and 54; both are the same.

Note:
Exercise:

Problem: Multiply:

ⓐ $8 \cdot 6$
ⓑ $6 \cdot 8$

Solution:

48 and 48; both are the same.

Multiplying by 10

Whenever we multiply with one factor a 10, the product has a 0 as the last digit and the preceding digits are the other factor. For example 10 x 4 = 40 and 4 x 10 = 40. Why do products like these always have a last digit of 0?

If we can find a reason for either one of these then the other must also be true because of the Commutative Property of Multiplication.
Thought of as repeated addition: 10 x 4 = 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4.

Doing the addition gives the following running total: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40.
This is both the 4 row and column of the multiplication table. This doesn't really help us understand why the last digit is 0.

But looking at 4 x 10 does: 10 + 10 + 10 + 10
This has the running total 10, 20, 30, 40. Every step of the way the total ended in 0 in the ones place. When the addend is 10 there is 0 in the ones place. Since all of the addends are 10, all of those zeros will still give a 0 in the ones place.
The tens place has exactly the other factor number of tens. In this case 4. So the tens place is 4, giving a final answer of 40.

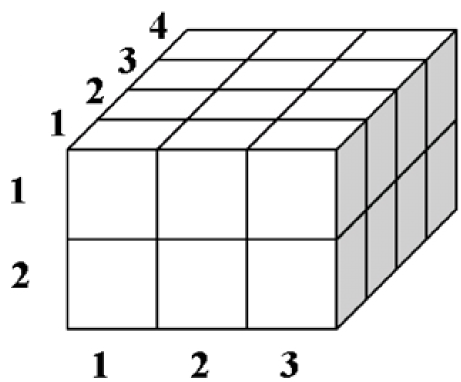
This works for larger numbers as well. For example 23 x 10 = 230. Adding twenty-three 10s will give 0 in the ones place. And twenty-three 1s in the tens place will cause the 2 to carry to the hundreds place and the 3 to remain in the tens place. In general, multiplying by 10 is easy because the digits of the product are the digits of the other factor with them all moved left to one higher place, and a 0 in the ones place.

Associative Property of Multiplication

Just like addition, multiplication is associative. That means that no matter how three factors are grouped, the result should be the same. This is claiming that:

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Here is an example. When a = 2, b = 3, and c = 4: (2 x 3) x 4 = 6 x 4 = 24 while 2 x (3 x 4) = 2 x 12 = 24. It works, at least for these numbers.



When we multiply the 2 by 3 we get 6. This is the number of cubes on just the front face of the picture. When that result multiplies 4, we are taking that entire row of 4 cubes and duplicating it 6 times. These rows of 4 are arranged in a 2 by 3 grid. This gives 24 cubes.

When we calculate 2 x (3 x 4), the first part is 3 x 4 which gives 12. For example, the 12 cubes on top. When we multiply that result by 2, that entire layer of 12 cubes is duplicated so that there are 2 levels each with 12 cubes. This also gives 24 cubes.

There was nothing special about the numbers 2, 3, and 4 other than they are small numbers so it was easy to illustrate. The idea works no matter how big the numbers. This supports the claim that the Associative Property of Multiplication is true in general. Notice that we had three factors, a, b, and c, for this property and that the illustration arranged the cubes (instead of counters) in a 3-dimensional grid. All we did was look at those cubes from different perspectives to see different ways to to calculate the total number which

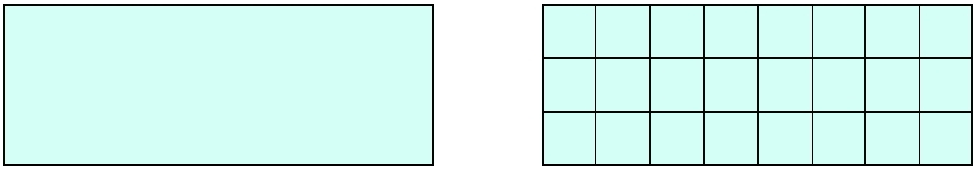
did not change. We did a similar thing in 2-dimensions for the commutative property when we looked at the counters after turning the array sideways.

Combined, the commutative and associative properties for multiplication give a result similar to the commutative and associative properties of addition. We can rearrange factors and multiply in any order we want, and we will get the same product. For example, 2 x 4 x 7 x 9 x 0 gives the same product as 0 x 2 x 4 x 7 x 9. Written the first way, we have to do 3 multiplications before finally multiplying by 0 and getting the answer 0. Written the second way we immediately see that the product is going to be 0 and thus have less work to do.

Area Model of Multiplication

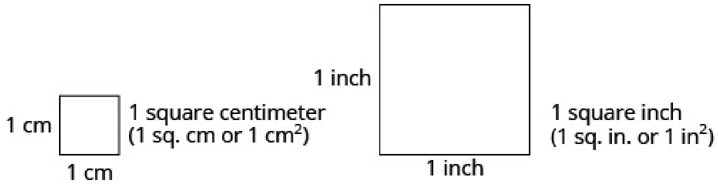
We can think of multiplication as the **area** of a rectangle rather than in terms of counters. The area is a measure of the amount of surface that is covered by the shape. It can be thought of as the number of unit squares that completely and exactly cover the figure with no overlap.

Consider a rectangle that is 8 centimeters by 3 centimeters. If we were to cover the figure with little unit squares we could think of the units as counters and the number needed would equal the length times the width.



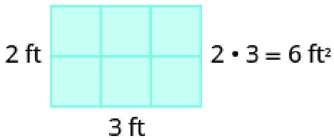
The area way of thinking about the rectangle and the counter way of thinking about the rectangle give us the same answer. Sometimes it will be easier to think in terms of area rather than having to think about counters. Other times counters will still be easier. In mathematics it is helpful to have more than one model so you can pick which ever one works best for a particular problem.

If we want to know the size of a wall that needs to be painted or a floor that needs to be carpeted, we will need to find its area. Area is measured in square units. All four sides are the same length for a square and the figure has four equal corners. Any unit of length can be used to make a square. We often use square inches, square feet, square centimeters, or square miles to measure area. A square centimeter is a square that is one centimeter (cm.) on a side. A square inch is a square that is one inch on each side, and so on.



For a rectangular figure, the area is the product of the length and the width. [link] shows a rectangular mat with a length of 2 feet and a width of 3 feet. Each square is 1 foot wide by 1 foot long, or 1 square foot. The mat can be covered completely with no overlap with 6 squares. The area of the mat is 6 square feet.

Because the squares are arranged in columns and rows of equal size, rather than count the squares or add the rows or columns, we can multiply to get the total amount. Therefore the area of a rectangle is its length x width: A = L x W.



The area of a rectangle is the product of its length and its width, or 6 square feet.

Distributive Property of Multiplication over Addition

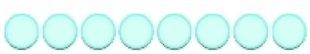
The Distributive Property of Multiplication over Addition is our first property that involves more than one operation. Normally it is just called the Distributive Property. The reason for the long name is that in more advanced math there are other distributive properties.

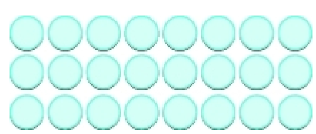
$a \cdot (b + c) = a \cdot b + a \cdot c$

To help get a feeling for why this property is true, we'll use some numbers to see how it works.
For example, let a=3, b=6, and c=2. Then the equation becomes: 3 x (6 + 2) = (3 x 6) + (3 x 2)

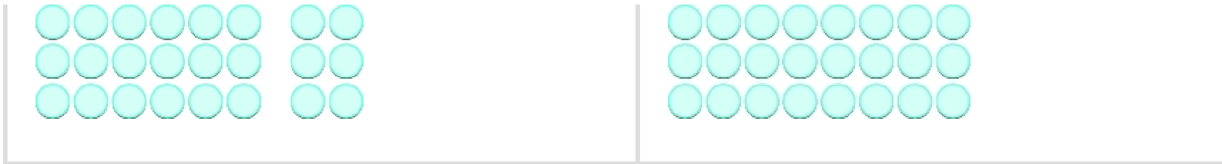
The left hand side of the equation computes: 3 x (6 + 2) = 3 x 8 = 24.
The right hand side of the equation computes: (3 x 6) + (3 x 2) = 18 + 6 = 24. At least in this case, the property is true.

We can use counters to illustrate each side of the equation. The left hand side of the equation has one row of six counters with two counters added to it making one row of 8 counters. This is then multiplied to make 3 rows of 8 counters.





The right hand side of the equation starts with 3 rows of 6 counters followed by 3 rows of 2 counters. They are then added resulting in 3 rows of 8 counters.



In both cases that we end up with 24 counters arranged as 3 rows of 8.

This works for any number of rows no matter how long the rows are as long as they are all same length.

Multiplying Multi-digit Numbers

With the exception of multiplying by 10, all of the multiplication we've done so far has been a single digit times a single digit. We have built a multiplication table that organizes and holds those facts and either we can recall them automatically or have a strategy for quickly recalling them.

What about multi-digit multiplication? Should we enlarge the multiplication table for numbers greater than 10? We could, but this isn't a good strategy because the table would quickly either become very, very large or else not be big enough.

Because our numbers are represented using place value this isn't necessary. We can master a procedure for multiplying large numbers that doesn't require learning any more facts than the basic ones we already know.

There is more than one correct way to do multi-digit multiplication. We will look at two related ways. The first way you might have learned in school. We'll call it the Compact Method or Standard Method. The second way uses the Area Model of Multiplication and so we'll call it the Area Method. It has the benefit of being easier to find your mistakes if you happen to make one. Let's look at the more common way first.

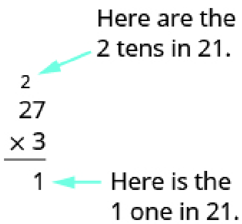
To multiply numbers with more than one digit, it is usually easier to write the numbers vertically in columns just as we did for addition and subtraction. It is also usually easier to write the smaller number with fewer digits on the bottom.
Equation:

$$\begin{array}{r} 27 \\ \underline{\times 3} \end{array}$$

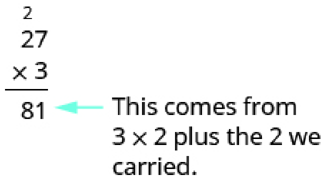
We start by multiplying 3 by 7.
Equation:

$$3\ \times\ 7 = 21$$

We write the 1 in the ones place of the product. We carry the 2 tens by writing 2 above the tens place.

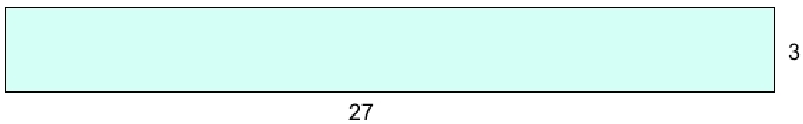


Then we multiply the 3 by the 2, and add the 2 above the tens place to the product. So $3 \times 2 = 6$, and $6 + 2 = 8$. Write the 8 in the tens place of the product.

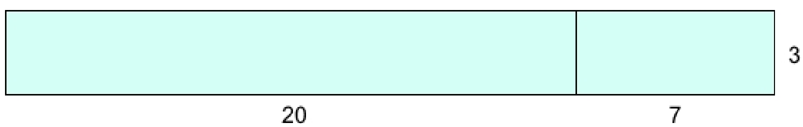


The product is 81.

The Area Model of Multiplication
Let's look at the same problem using the Area Model of Multiplication. Start by drawing a 27 by 3 rectangle. Occasionally, when we draw diagrams in math it helps to have the sizes fairly accurate. This is NOT one of those times, so don't worry about having the lengths perfect in relationship to each other.



The key to this method is to split any side that is has more than one digit into parts based on the Expanded Form for the number. In our example, we split 27 into 20 and 7. Because it is more familiar for us to have the 20 on the left and the 7 on the right, it is easiest if we do it that way. The other dimension, 3, is unchanged. Now we have two smaller rectangles rather than just one big rectangle.



We can find 27 x 3 if we can find the area of the 27 by 3 rectangle.
We can find the area of the large rectangle by finding the area of the two smaller rectangles and then adding them together.
This uses Expanded Form and then the Distributive Property.

3 x 27 = 3 x (20 + 7) = (3 x 20) + (3 x 7).

Calculating 3 x 7 is easy, it is just a basic multiplication fact. Calculating 3 x 20 requires a little more work.
You may already know that 3 x 20 = 60.
Let's see how we know that using the properties of multiplication and the multiplication table.

Any number whose last digit is a "0" is the product of the number that has all the same digits in the same order but without the final "0" times 10.
In this case, 20 = 2 x 10. "2" has the same digits as "20" except for the final "0".

Substituting 2 x 10 for 20 gives: 3 x (2 x 10).
Using the Associative Property of Multiplication yields: (3 x 2) x 10.
Using the multiplication table gives: 6 x 10.
Finally using the Multiplication Property of 10 gives 60.

To finish the problem, we need to calculate 60 + 21 = 81.

No need to draw a diagram to multiply larger numbers. This illustration that shows why the method works. It is recommended that you write:

$$\begin{array}{r} 27 \\ \times\ 3 \\ \hline 21 \\ +60 \\ \hline 81 \end{array}$$

This is only a tiny bit more writing than the compact method. It has an advantage of separating the recall of multiplication facts from the addition.
Each multiplication is written down and then addition is performed. This makes it easier to check your work.
It also shows the connection between those products and the smaller rectangles that make up the large rectangle.
In our example, we see the 3 x 7 = 21 rectangle and the 3 x 20 = 60 rectangle combine to make the 3 x 27 = 81 rectangle.

Example:
Exercise:

Problem: Multiply: $15 \cdot 4$.

Solution:
Compact Method

Write the numbers so the digits 5 and 4 line up vertically.	$\begin{array}{r} 15 \\ \times\ 4 \\ \hline \end{array}$
Multiply 4 by the digit in the ones place of 15. $4 \cdot 5 = 20$.	
Write 0 in the ones place of the product and carry the 2 tens.	$\begin{array}{r} \overset{2}{1}5 \\ \times\ 4 \\ \hline 0 \end{array}$
Multiply 4 by the digit in the tens place of 15. $4 \cdot 1 = 4$. Add the 2 tens we carried. $4 + 2 = 6$.	
Write the 6 in the tens place of the product.	$\begin{array}{r} \overset{2}{1}5 \\ \times\ 4 \\ \hline 60 \end{array}$

Try to do this problem again using the area method. What large rectangle would you draw? What two rectangles would you break it up into?
Using the area model, what numbers should you write and how should they be organized?

Note:
For this problem, think about the dimensions of the two small rectangles but don't draw them. Instead write their areas and add them for the final answer.
Exercise:

Problem: Multiply: $64 \cdot 8$.
One rectangle is 4 x 8 = 32 and the other is 60 x 8 = 480.

Solution:

512

Note:
Exercise:

Problem: Multiply: $57 \cdot 6$.

Solution:

342

Example:
Exercise:

Problem: Multiply: $286 \cdot 5$.

Solution:
Compact Method

Write the numbers so the digits 5 and 6 line up vertically.	$\begin{array}{r} 286 \\ \underline{\times\ 5} \end{array}$
Multiply 5 by the digit in the ones place of 286. $5 \cdot 6 = 30$.	
Write the 0 in the ones place of the product and carry the 3 to the tens place.Multiply 5 by the digit in the tens place of 286. $5 \cdot 8 = 40$.	

	$\begin{array}{r} \overset{3}{2}86 \\ \times\ 5 \\ \hline 0 \end{array}$
Add the 3 tens we carried to get $40 + 3 = 43$. Write the 3 in the tens place of the product and carry the 4 to the hundreds place.	$\begin{array}{r} \overset{4}{2}\overset{3}{8}6 \\ \times\ 5 \\ \hline 30 \end{array}$
Multiply 5 by the digit in the hundreds place of 286. $5 \cdot 2 = 10$. Add the 4 hundreds we carried to get $10 + 4 = 14$. Write the 4 in the hundreds place of the product and the 1 to the thousands place.	$\begin{array}{r} \overset{4}{2}\overset{3}{8}6 \\ \times\ 5 \\ \hline 1{,}430 \end{array}$

Do this problem using the area model. Since there are 3 digits in the larger number, the large rectangle will be broken up into three smaller rectangles. Draw them and then complete the multiplication. Hint #1: Use the Expanded Form for 286 = 200 + 80 + 6. Hint #2: 200 = 20 x 10 = 2 x 10 x 10.

Note:
Exercise:

Problem: Multiply: $347 \cdot 5$.

Solution:

1,735

Note:
Exercise:

Problem: Multiply: $462 \cdot 7$.

Solution:

3,234

When we multiply by a number with two or more digits, we multiply by each of the digits separately, working from right to left. Each separate product of the digits is called a partial product. When we write partial products, we must make sure to line up the place values.

Example:
Exercise:

Problem: Multiply:

ⓐ $47 \cdot 10$
ⓑ $47 \cdot 100$.

Solution:
Solution

When we multiply 47 times 10, the product is 470. Notice that 10 has one zero, and we put the zero directly to the right of the 47 to get the product. When we multiply 47 times 100, the product is 4,700. Notice that 100 has two zeros and we put two zeros after 47 to get the product.

Do you see the pattern? If we multiplied 47 times 10,000, which has four zeros, we would put four zeros after 47 to get the product 470,000.

Why does this work?
47 x 10,000 = 47 x (10 x 1,000)
47 x (10 x 1,000) = (47 x 10) x 1,000
(47 x 10) x 1,000 = 470 x 1,000
470 x 1,000 = 470 x (10 x 100)
470 x (10 x 100) = (470 x 10) x 100
(470 x 10) x 100 = 4,700 x 100
4,700 x 100 = 4,700 x (10 x 10)
4,700 x (10 x 10) = (4,700 x 10) x 10
(4,700 x 10) x 10 = 47,000 x 10
47,000 x 10 = 470,000

This repeatedly uses the Associative Property of Multiplication and our rule for Multiplying by 10.

You do not have to think of this process every time to get the correct answer.
Use the shortcut of counting the total number of zeros to the right of the first non-zero digits of both factors. For example, 210 x 300 = 63,000 because 21 x 3 = 63 and there are a total of 3 zeros prior to the non-zero digits.

Note:
Exercise:

Problem: Multiply:

ⓐ $54 \cdot 10$
ⓑ $54 \cdot 100$.

Solution:

ⓐ 540
ⓑ 5,400

Note:
Exercise:

Problem: Multiply:

ⓐ $75 \cdot 10$
ⓑ $75 \cdot 100.$

Solution:

ⓐ 750
ⓑ 7,500

When there are three or more factors, we multiply the first two and then multiply their product by the next factor. For example:

to multiply	$8 \cdot 3 \cdot 2$
first multiply $8 \cdot 3$	$24 \cdot 2$
then multiply $24 \cdot 2.$	48

Since multiplication is commutative and associative, we could multiply the factors in any order and get the same answer. It might be easier to multiply the 3 x 2 = 6 first and then 8 x 6 = 48 since all of these multiplications are single digit facts.

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation into words. Now we'll reverse the process and translate word phrases into math notation. Some of the words that indicate multiplication are given in [link].

Operation	**Word Phrase**	**Example**	**Expression**
Multiplication	times product twice	3 times 8 the product of 3 and 8 twice 4	$3 \times 8, 3 \cdot 8, (3)(8),$ $(3)8,$ or $3(8)$ $2 \cdot 4$

Example:
Exercise:

Problem: Translate and simplify: the product of 12 and 4.

Solution:
Solution

The word *product* tells us to multiply. The words *of* 12 *and* 4 tell us the two factors.

	the product of 12 and 4
Translate.	$12 \cdot 4$
Multiply.	48

Note:
Exercise:

Problem: Translate and simplify the product of 13 and 8.

Solution:

$13 \cdot 8$; 104

Note:
Exercise:

Problem: Translate and simplify the product of 47 and 5.

Solution:

$47 \cdot 5$; 235

Example:
Exercise:

Problem: Translate and simplify: twice two hundred eleven.

Solution:
Solution

The word *twice* tells us to multiply by 2.

	twice two hundred eleven
Translate.	2(211)
Multiply.	422

Note:
Exercise:

Problem: Translate and simplify: twice one hundred sixty-seven.

Solution:

2(167); 334

Note:
Exercise:

Problem: Translate and simplify: twice two hundred fifty-eight.

Solution:

2(258); 516

Multiply Whole Numbers in Applications

We will use the same strategy we used previously to solve applications of multiplication.

- Determine what we are asked to find.
- Write a phrase that gives the information to find it.
- Translate the phrase into math notation.
- Simplify to get the answer.
- Write a sentence to answer the question, using the appropriate units.

Example:
Exercise:

Problem:

Humberto bought 4 sheets of stamps. Each sheet had 20 stamps. How many stamps did Humberto buy?

Solution:
Solution

We are asked to find the total number of stamps.

Write a phrase for the total.	the product of 4 and 20
Translate to math notation.	$4 \cdot 20$
Multiply.	$\begin{array}{r} 20 \\ \times 4 \\ \hline 80 \end{array}$
Write a sentence to answer the question.	Humberto bought 80 stamps.

Note:
Exercise:

Problem:

Valia donated water for the snack bar at her son's baseball game. She brought 6 cases of water bottles. Each case had 24 water bottles. How many water bottles did Valia donate?

Solution:

Valia donated 144 water bottles.

Note:
Exercise:

Problem:

Vanessa brought 8 packs of hot dogs to a family reunion. Each pack has 10 hot dogs. How many hot dogs did Vanessa bring?

Solution:

Vanessa bought 80 hot dogs.

Example:
Exercise:

Problem:

When Rena cooks rice, she uses twice as much water as rice. How much water does she need to cook 4 cups of rice?

Solution:
Solution

We are asked to find how much water Rena needs.

Write as a phrase.	twice as much as 4 cups
Translate to math notation.	$2 \cdot 4$
Multiply to simplify.	8
Write a sentence to answer the question.	Rena needs 8 cups of water for cups of rice.

Note:
Exercise:

Problem:

Erin is planning her flower garden. She wants to plant twice as many dahlias as sunflowers. If she plants 14 sunflowers, how many dahlias does she need?

Solution:

Erin needs 28 dahlias.

Note:
Exercise:

Problem:

A college choir has twice as many women as men. There are 18 men in the choir. How many women are in the choir?

Solution:

There are 36 women in the choir.

Example:
Exercise:

Problem:

Van is planning to build a patio. He will have 8 rows of tiles, with 14 tiles in each row. How many tiles does he need for the patio?

Solution:
Solution

We are asked to find the total number of tiles.

Write a phrase.	the product of 8 and 14
Translate to math notation.	$8 \cdot 14$
Multiply to simplify.	$\begin{array}{r} \overset{3}{1}4 \\ \underline{\times 8} \\ 112 \end{array}$
Write a sentence to answer the question.	Van needs 112 tiles for his patio.

Note:
Exercise:

Problem:

Jane is tiling her living room floor. She will need 32 rows of tile, with 10 tiles in each row. How many tiles does she need for the living room floor?

Solution:

Jane needs 320 tiles.

Note:
Exercise:

Problem:

Yousef is putting shingles on two garage roofs. Each roof needs 10 rows of shingles, with 54 shingles in each row. How many shingles does he need in total?

Solution:

Yousef needs 1,080 tiles.

Example:
Exercise:

Problem:

Jen's kitchen ceiling is a rectangle that measures 9 feet long by 12 feet wide. What is the area of Jen's kitchen ceiling?

Solution:
Solution

We are asked to find the area of the kitchen ceiling.

Write a phrase for the area.	the product of 9 and 12
Translate to math notation.	$9 \cdot 12$
Multiply.	$\begin{array}{r} \overset{1}{1}2 \\ \underline{\times 9} \\ 108 \end{array}$
Answer with a sentence.	The area of Jen's kitchen ceiling is 108 square feet.

Note:
Exercise:

Problem:

Zoila bought a rectangular rug. The rug is 8 feet long by 5 feet wide. What is the area of the rug?

Solution:

The area of the rug is 40 square feet.

Note:
Exercise:

Problem:

Rene’s driveway is a rectangle 15 yards long by 7 yards wide. What is the area of the driveway?

Solution:

The area of the driveway is 105 square yards.

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Area
- Multiplying Whole Numbers
- Multiplication with Partial Products
- Example of Multiplying by Whole Numbers

Key Concepts

Operation	Notation	Expression	Read as	Result
Multiplication	$\times$ $\cdot$ $(\ )$	3×8 $3 \cdot 8$ $3(8)$	three times eight	the product of 3 and 8

- **Multiplication Property of Zero**

- The product of any number and 0 is 0.
  $a \cdot 0 = 0$
  $0 \cdot a = 0$

- **Identity Property of Multiplication**
  - The product of any number and 1 is the number.
    $1 \cdot a = a$
    $a \cdot 1 = a$
- **Commutative Property of Multiplication**
  - Changing the order of the factors does not change their product.
    $a \cdot b = b \cdot a$
- **Multiplying by 10**
  - Multiplying by a factor of 10 results in the same digits immediately followed by a digit of zero.
- **Associative Property of Multiplication**
  - Changing the grouping of the factors does not change their product.
    $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
- **Distributive Property of Multiplication over Addition**
  - A factor multiplying a sum can be changed to the sum of the factor multiplying each addend.
    $a \cdot (b + c) = a \cdot b + a \cdot c$

Exercises

Practice Makes Perfect

Use Multiplication Notation

In the following exercises, translate from math notation to words.

Exercise:

Problem: 4×7

Solution:

four times seven; the product of four and seven

Exercise:

Problem: 8×6

Exercise:

Problem: $5 \cdot 12$

Solution:

five times twelve; the product of five and twelve

Exercise:

Problem: $3 \cdot 9$

Exercise:

Problem: $(10)(25)$

Solution:

ten times twenty-five; the product of ten and twenty-five

Exercise:

Problem: $(20)(15)$

Exercise:

Problem: $42(33)$

Solution:

forty-two times thirty-three; the product of forty-two and thirty-three

Exercise:

Problem: $39(100)$

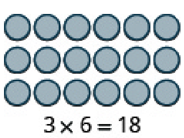
Model Multiplication of Whole Numbers

In the following exercises, model the multiplication.

Exercise:

Problem: 3×6

Solution:



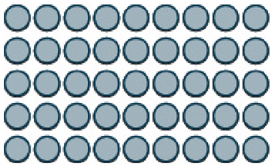
Exercise:

Problem: 4×5

Exercise:

Problem: 5×9

Solution:



5 × 9 = 45

Exercise:

Problem: 3×9

Multiply Whole Numbers

In the following exercises, fill in the missing values in each chart.

Exercise:

Problem:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0		0	0	0	0		0
1	0	1	2	3			6	7	8	
2		2	4	6	8		12			18
3	0		6		12	15		21		27
4	0	4			16	20		28	32	
5	0	5	10	15			30		40	
6	0	6	12		24			42		54
7			14	21		35			56	63
8	0	8		24			48		64	
9	0	9	18		36	45			72	

Solution:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Exercise:

Problem:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0		0		0	0
1	0	1	2		4	5		7		9
2	0		4		8	10		14	16	
3		3		9			18		24	
4	0	4	8	12			24	28		36
5	0	5		15	20		30	35	40	
6			12	18			36	42		54
7	0	7		21		35			56	63
8	0	8	16		32		48		64	72
9			18	27	36			63		

Exercise:

Problem:

×	3	4	5	6	7	8	9
4							
5							
6							
7							
8							
9							

Solution:

×	3	4	5	6	7	8	9
4	12	16	20	24	28	32	36
5	15	20	25	30	35	40	45
6	18	24	30	36	42	48	54
7	21	28	35	42	49	56	63
8	24	32	40	48	56	64	72
9	27	36	45	54	63	72	81

Exercise:

Problem:

×	4	5	6	7	8	9
3						
4						
5						
6						
7						
8						
9						

Exercise:

Problem:

×	3	4	5	6	7	8	9
6							
7							
8							
9							

Solution:

×	3	4	5	6	7	8	9
6	18	24	30	36	42	48	54
7	21	28	35	42	49	56	63
8	24	32	40	48	56	64	72
9	27	36	45	54	63	72	81

Exercise:

Problem:

×	6	7	8	9
3				
4				
5				
6				
7				
8				
9				

Exercise:

Problem:

×	5	6	7	8	9
5					
6					
7					
8					
9					

Solution:

×	5	6	7	8	9
5	25	30	35	40	45
6	30	36	42	48	54
7	35	42	49	56	63
8	40	48	56	64	72
9	45	54	63	72	81

Exercise:

Problem:

×	6	7	8	9
6				
7				
8				
9				

In the following exercises, multiply.

Exercise:

Problem: $0 \cdot 15$

Solution:

0

Exercise:

Problem: $0 \cdot 41$

Exercise:

Problem: $(99)0$

Solution:

0

Exercise:

Problem: $(77)0$

Exercise:

Problem: $1 \cdot 43$

Solution:

43

Exercise:

Problem: $1 \cdot 34$

Exercise:

Problem: $(28)1$

Solution:

28

Exercise:

Problem: $(65)1$

Exercise:

Problem: $1(240{,}055)$

Solution:

240,055

Exercise:

Problem: $1(189{,}206)$

Exercise:

Problem:

ⓐ $7 \cdot 6$
ⓑ $6 \cdot 7$

Solution:

ⓐ 42
ⓑ 42

Exercise:

Problem:

ⓐ 8×9
ⓑ 9×8

Exercise:

Problem: $(79)(5)$

Solution:

395

Exercise:

Problem: $(58)(4)$

Exercise:

Problem: $275 \cdot 6$

Solution:

$1{,}650$

Exercise:

Problem: $638 \cdot 5$

Exercise:

Problem: $3{,}421\ \times\ 7$

Solution:

$23{,}947$

Exercise:

Problem: $9{,}143\ \times\ 3$

Exercise:

Problem: $23 \cdot 10$

Solution:

230

Exercise:

Problem: $19 \cdot 10$

Exercise:

Problem: $(100)(36)$

Solution:

$3{,}600$

Exercise:

Problem: $(100)(25)$

Exercise:

Problem: $1{,}000(88)$

Solution:

$88{,}000$

Exercise:

Problem: $1{,}000(46)$

Exercise:

Problem: $50 \times 1,000,000$

Solution:

50,000,000

Exercise:

Problem: $30 \times 1,000,000$

Translate Word Phrases to Math Notation

In the following exercises, translate and simplify.

Exercise:

Problem: the product of 18 and 3

Solution:

$18 \cdot 3$; 54

Exercise:

Problem: the product of 5 and 22

Exercise:

Problem: fifty-one times seven

Solution:

51(7); 357

Exercise:

Problem: forty-eight times seven

Exercise:

Problem: twice 249

Solution:

2(249); 498

Exercise:

Problem: twice 589

Exercise:

Problem: ten times three hundred seventy-five

Solution:

10(375); 3,750

Exercise:

Problem: ten times two hundred fifty-five

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: 8×37

Solution:

296

Exercise:

Problem: 86×9

Exercise:

Problem: $415 - 267$

Solution:

148

Exercise:

Problem: $341 - 285$

Exercise:

Problem: $6{,}251 + 4{,}749$

Solution:

11,000

Exercise:

Problem: $3{,}816 + 8{,}184$

Exercise:

Problem: $(6)(204)$

Solution:

1,224

Exercise:

Problem: $(3)(801)$

Exercise:

Problem: $947 \cdot 0$

Solution:

0

Exercise:

Problem: $947 + 0$

Exercise:

Problem: $15{,}382 + 1$

Solution:

15,383

Exercise:

Problem: $15{,}382 \cdot 1$

In the following exercises, translate and simplify.

Exercise:

Problem: the difference of 50 and 18

Solution:

50 − 18; 32

Exercise:

Problem: the difference of 90 and 66

Exercise:

Problem: twice 35

Solution:

2(35); 70

Exercise:

Problem: twice 140

Exercise:

Problem: 20 more than 980

Solution:

20 + 980; 1,000

Exercise:

Problem: 65 more than 325

Exercise:

Problem: the product of 8 and 875

Solution:

8(875); 7,000

Exercise:

Problem: the product of 5 and 905

Exercise:

Problem: subtract 74 from 89

Solution:

89 − 74; 15

Exercise:

Problem: subtract 45 from 99

Exercise:

Problem: the sum of 3,075 and 950

Solution:

3,075 + 950; 4,025

Exercise:

Problem: the sum of 6,308 and 724

Exercise:

Problem: 366 less than 814

Solution:

814 − 366; 448

Exercise:

Problem: 388 less than 925

Multiply Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Party supplies Tim brought 9 six-packs of soda to a club party. How many cans of soda did Tim bring?

Solution:

Tim brought 54 cans of soda to the party.

Exercise:

Problem:

Sewing Kanisha is making a quilt. She bought 6 cards of buttons. Each card had four buttons on it. How many buttons did Kanisha buy?

Exercise:

Problem:

Field trip Seven school busses let off their students in front of a museum in Washington, DC. Each school bus had 44 students. How many students were there?

Solution:

There were 308 students.

Exercise:

Problem:

Gardening Kathryn bought 8 flats of impatiens for her flower bed. Each flat has 24 flowers. How many flowers did Kathryn buy?

Exercise:

Problem:

Charity Rey donated 7 twelve-packs of t-shirts to a homeless shelter. How many t-shirts did he donate?

Solution:

Rey donated 84 t-shirts.

Exercise:

Problem:

School There are 28 classrooms at Anna C. Scott elementary school. Each classroom has 8 computers. What is the total number of computers?

Exercise:

Problem:

Recipe Stephanie is making punch for a party. The recipe calls for twice as much fruit juice as club soda. If she uses 10 cups of club soda, how much fruit juice should she use?

Solution:

Stephanie should use 20 cups of fruit juice.

Exercise:

Problem:

Gardening Hiroko is putting in a vegetable garden. He wants to have twice as many lettuce plants as tomato plants. If he buys 12 tomato plants, how many lettuce plants should he get?

Exercise:

Problem:

Government The United States Senate has twice as many senators as there are states in the United States. There are 50 states. How many senators are there in the United States Senate?

Solution:

There are 100 senators in the U.S. senate.

Exercise:

Problem:

Recipe Andrea is making potato salad for a buffet luncheon. The recipe says the number of servings of potato salad will be twice the number of pounds of potatoes. If she buys 30 pounds of potatoes, how many servings of potato salad will there be?

Exercise:

Problem:

Painting Jane is painting one wall of her living room. The wall is rectangular, 13 feet wide by 9 feet high. What is the area of the wall?

Solution:

The area of the wall is 117 square feet.

Exercise:

Problem:

Home décor Shawnte bought a rug for the hall of her apartment. The rug is 3 feet wide by 18 feet long. What is the area of the rug?

Exercise:

Problem: **Dog run** A dog run is 3 feet wide by 10 feet long. What is the area of the dog run?

Solution:

The area of the dog run is 30 square feet.

Exercise:

Problem:

Gardening June has a vegetable garden. The garden is rectangular, with length 12 yards and width 8 feet. What is the area of the garden?

Exercise:

Problem:

Screen size A computer screen is 16 inches long by 9 inches wide. What is the area of the computer screen?

Solution:

The area of the computer screen is 144 square inches.

Exercise:

Problem:

Red carpet The standard size red carpet for dignitaries is 25 feet by 2 feet. What is the area of the red carpet?

Everyday Math

Exercise:

Problem:

Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price rose $9 per share. How much money did Javier's portfolio gain?

Solution:

Javier's portfolio gained $2,700.

Exercise:

Problem:

Salary Carlton got a $100 raise in each paycheck. He gets paid 24 times a year. How much higher is his new annual salary?

Writing Exercises

Exercise:

Problem:

How confident do you feel about your knowledge of the multiplication facts? If you are not fully confident, what will you do to improve your skills?

Solution:

Answers will vary.

Exercise:

Problem: How have you used models to help you learn the multiplication facts?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use multiplication notation.			
model multiplication of whole numbers.			
multiply whole numbers.			
translate word phrases to math notation.			
multiply whole numbers in applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

product
: The product is the result of multiplying two or more numbers.

Divide Whole Numbers Beginning Level
By the end of this section, you will be able to:

- Use division notation
- Model division of whole numbers
- Divide whole numbers
- Translate word phrases to math notation
- Divide whole numbers in applications

Note:
Before you get started, take this readiness quiz.

1. Multiply: $27 \cdot 3$.
   If you missed this problem, review [link].
2. Subtract: $43 - 26$.
   If you missed this problem, review [link]

Use Division Notation

So far we have explored addition, subtraction, and multiplication. Now let’s consider division. Suppose you have the 12 cookies in [link] and want to package them in bags with 4 cookies in each bag. How many bags would we need?

You might put 4 cookies in first bag, 4 in the second bag, and so on until you run out of cookies. Doing it this way, you would fill 3 bags.

In other words, starting with the 12 cookies, you would take away, or subtract, 4 cookies at a time. Division is a way to represent repeated subtraction just as multiplication represents repeated addition.

Instead of subtracting 4 repeatedly, we can write
Equation:

$$12 \div 4$$

We read this as *twelve divided by four* and the result is the **quotient** of 12 and 4. The quotient is 3 because we can subtract 4 from 12 exactly 3 times. We call the number being divided the **dividend** and the number dividing it the **divisor**. In this case, the dividend is 12 and the divisor is 4.

The word "quotient" comes from Latin and means "how many times".

In the past you may have used the notation $4\overline{)12}$, but this division also can be written as $12 \div 4,\ 12/4,\ \frac{12}{4}$. In each case the 12 is the dividend and the 4 is the divisor.

Note:
Operation Symbols for Division
To represent and describe division, we can use symbols and words.

Operation	**Notation**	**Expression**	**Read as**	**Result**
Division	$\div$ $\frac{a}{b}$ $b\overline{)a}$ a/b	$12 \div 4$ $\frac{12}{4}$ $4\overline{)12}$ $12/4$	Twelve divided by four	the quotient of 12 and 4

Division is performed on two numbers at a time. When translating from math notation to English words, or English words to math notation, look for the words *of* and *and* to identify the numbers.

Example:
Exercise:

Problem: Translate from math notation to words.

ⓐ $64 \div 8$ ⓑ $\frac{42}{7}$ ⓒ $4\overline{)28}$

Solution:
Solution

- ⓐ We read this as *sixty-four divided by eight* and the result is *the quotient of sixty-four and eight*.
- ⓑ We read this as *forty-two divided by seven* and the result is *the quotient of forty-two and seven*.
- ⓒ We read this as *twenty-eight divided by four* and the result is *the quotient of twenty-eight and four*.

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $84 \div 7$ ⓑ $\frac{18}{6}$ ⓒ $8\overline{)24}$

Solution:

- ⓐ eighty-four divided by seven; the quotient of eighty-four and seven
- ⓑ eighteen divided by six; the quotient of eighteen and six.
- ⓒ twenty-four divided by eight; the quotient of twenty-four and eight

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $72 \div 9$ ⓑ $\frac{21}{3}$ ⓒ $6\overline{)54}$

Solution:

- ⓐ seventy-two divided by nine; the quotient of seventy-two and nine
- ⓑ twenty-one divided by three; the quotient of twenty-one and three
- ⓒ fifty-four divided by six; the quotient of fifty-four and six

Model Division of Whole Numbers

As we did with multiplication, we will model division using counters. The operation of division helps us organize items into equal groups as we start with the number of items in the dividend and subtract the number in the divisor repeatedly.

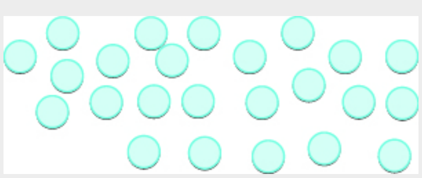
Example:
Exercise:

Problem: Model the division: $24 \div 8$.

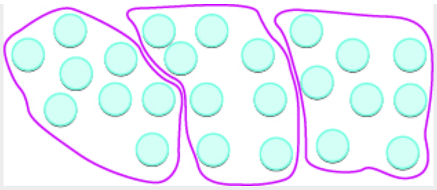
Solution:
Solution

To find the quotient $24 \div 8$, we want to know how many groups of 8 are in 24.

Model the dividend. Start with 24 counters.



The divisor tell us the number of counters we want in each group. Form groups of 8 counters.



Count the number of groups. There are 3 groups.

$24 \div 8 = 3$

Division as Repeated Subtraction

We can think of division as repeated subtraction where we are trying to find out how many times the divisor can be subtracted from the dividend.

$24 - 8 = 16$
$16 - 8 = 8$
$8 - 8 = 0$

And the quotient (answer) is 3 because 8 could be subtracted 3 times from 24.

Division and Multiplication are Opposites

Addition is the opposite of subtraction. Repeated subtraction can be undone by repeated addition.

$0 + 8 = 8$
$8 + 8 = 16$
$16 + 8 = 24$

Repeated addition can be replaced by multiplication. That makes division and multiplication opposites.

$24 \div 8 = 3$
$3 \cdot 8 = 24$

Another way to think of Division

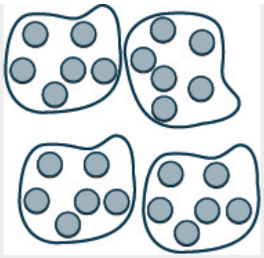
Another way to think of dividing 24 by 8 is to think of having 24 counters and splitting it evenly for 8 people. The question is how many counters will each person get. Here we want to know the size of the portion rather than how many portions of a particular size can we make.

Before we were thinking, how many groups of 8 counters can we make with 24 counters: $\mathrm{a} \cdot 8 = 24$
Now we are thinking, how big will the groups be if I make 8 groups using 24 counters: $8 \cdot \mathrm{a} = 24$
The commutative property of multiplication tells us that the order of the factors doesn't matter, so we can interpret division either way. In both cases a = 3.

Note:
Exercise:

Problem: Model: $24 \div 6$.

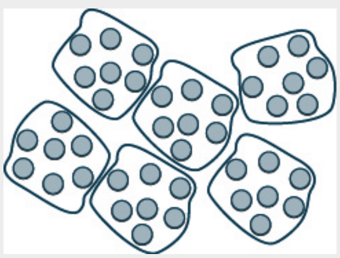
Solution:



Note:
Exercise:

Problem: Model: $42 \div 7$.

Solution:

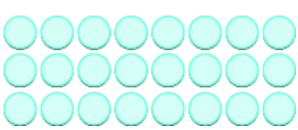


Divide Whole Numbers

We said that addition and subtraction are inverse operations because one undoes the other. Similarly, division is the inverse operation of multiplication. We know $12 \div 4 = 3$ because $3 \cdot 4 = 12$. Knowing all the multiplication number facts is very helpful when doing division.

We check our answer to division by multiplying the quotient by the divisor to determine if it equals the dividend. In [link], we know $24 \div 8 = 3$ is correct because $3 \cdot 8 = 24$.

If we arrange the counters of each group in a row and put the rows one on top of the other, then we can better see the relationship to multiplication.



This gives us a related way to understand division. We can think of division as making a rectangle where we know the area and the length of one side and we want to find the length of the side we do not yet know.

Example:
Exercise:

Problem: Divide. Then check by multiplying. ⓐ $42 \div 6$ ⓑ $\frac{72}{9}$ ⓒ $7\overline{)63}$

Solution:
Solution

ⓐ	
	mtd
Divide 42 by 6.	7
Check by multiplying. $7 \cdot 6$	
$42\checkmark$	

ⓑ	
	$\frac{72}{9}$
Divide 72 by 9.	8
Check by multiplying. $8 \cdot 9$	
$72\checkmark$	

ⓒ	
	$7\overline{)63}$
Divide 63 by 7.	9
Check by multiplying.	

$9 \cdot 7$	
$63 \checkmark$	

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $54 \div 6$ ⓑ $\frac{27}{9}$

Solution:

ⓐ 9 ⓑ 3

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $\frac{36}{9}$ ⓑ $8\overline{)40}$

Solution:

ⓐ 4 ⓑ 5

What is the quotient when you divide a number by itself?
Equation:

$$\frac{15}{15} = 1 \text{ because } 1 \cdot 15 = 15$$

Dividing any number $(\text{except } 0)$ by itself produces a quotient of 1. Also, any number divided by 1 produces a quotient of that number. These two ideas are stated in the Division Properties of One.

Note:
Division Properties of One

Any number (except 0) divided by itself is one.	$a \div a = 1$
Any number divided by one is the same number.	$a \div 1 = \mathrm{a}$

Example:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $11 \div 11$
ⓑ $\frac{19}{1}$
ⓒ $1\overline{)7}$

Solution:
Solution

ⓐ	
	$11 \div 11$
A number divided by itself is 1.	1
Check by multiplying. $1 \cdot 11$	
11✓	

ⓑ	
	$\frac{19}{1}$
A number divided by 1 equals itself.	19
Check by multiplying. $19 \cdot 1$	
19✓	

-

ⓒ	
	$1\overline{)7}$
A number divided by 1 equals itself.	7
Check by multiplying. $7 \cdot 1$	
$7\checkmark$	

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $14 \div 14$ ⓑ $\frac{27}{1}$

Solution:

ⓐ 1
ⓑ 27

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $\frac{16}{1}$ ⓑ $1\overline{)4}$

Solution:

ⓐ 16
ⓑ 4

Note:
Division Properties of Zero
Zero divided by any number other than zero is zero
Suppose we have $0, and want to divide it among 3 people. How much would each person get? Each person would get $0. Zero divided by any number other than zero is 0.

We can also understand this using the area model of division.
We want to know the length of the other side of a rectangle if one side is 3 units and the area is 0 square units.

Any length other than 0 units would result in an area greater than 0 square units but 3 x 0 = 0 so the length must be 0 units.
To use the area model, we think of the 3 units by 0 units as a "degenerate" rectangle. Here that means one of the dimensions has shrunk to 0.
Dividing by zero is undefined
Now suppose that we want to divide \$10 by 0people.
Thought of as repeated subtraction, this is asking how many groups can I form if I give each group \$0. When I subtract \$0 from \$10 I still have \$10 so this process never ends so there is no answer.

We can also understand this using the area model of division.
We want to know the length of the other side of a rectangle if one side is 0 units and the area is 10 square units.
Any length times 0 units results in an area of 0 square units so there is no length that gives 10 square units.

Dividing zero by zero also makes no sense and is also undefined. Thought of as multiplication it is $0 \text{ x } a = 0$. This is true for any value of a and thus there is no unique answer.

This is why dividing any number by 0 is undefined.

Zero divided by any number other than zero is 0.	$0 \div a = 0$
Dividing a number by zero is undefined.	$a \div 0$ undefined

Example:
Exercise:

Problem: Divide. Check by multiplying: ⓐ $0 \div 3$ ⓑ $10/0$.

Solution:
Solution

-
ⓐ	
	$0 \div 3$
Zero divided by any number is zero.	0
Check by multiplying. $0 \cdot 3$	
0✓	

ⓑ	
	$10/0$
Division by zero is undefined.	undefined

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $0 \div 2$ ⓑ $17/0$

Solution:

ⓐ 0 ⓑ undefined

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $0 \div 6$ ⓑ $13/0$

Solution:

ⓐ 0 ⓑ undefined

No Commutative Property of Division
There is not a commutative property for division. All it takes is one counterexamples to show this.
$2 \div 1 = 2$ while $1 \div 2 = \frac{1}{2}$. For example, $2 is not the same as half a dollar.

No Associative Property of Division
There is not an associative property for division. Consider $(8 \div 4) \div 2$ versus $8 \div (4 \div 2)$
$8 \div 4 = 2$ and $2 \div 2 = 1$
While $4 \div 2 = 2$ and $8 \div 2 = 4$
This shows that there is no associative property for division because 1 is not equal to 4.

Distributive Property of Division

This property is $(a + b) \div c = a \div c + b \div c$

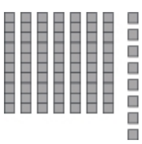
Division can involve fractions. Here is an example that avoids fractions.
$(18 + 6) \div 3 = 18 \div 3 + 6 \div 3$
$18 + 6 = 24$ and $24 \div 3 = 8$
While $18 \div 3 = 6$ and $6 \div 3 = 2$, and $6 + 2 = 8$. So they compute the same value.

Think of counters to be divide up evenly. You can add them together first and then divide them or you can divide them separately and then add those results together. The result is the same.

Another way to think of this is with money. Imagine you and a friend did two small jobs together but you did most of the work. So instead of splitting the money evenly, both of you agreed that your friend would only get one-third of the pay. One job paid $18 and the other paid $6. Added together that is $24. One-third of that is $8. If you paid him one-third after the first job that would be $6. And then for the second job $2 more for a total of $8. Either way the final amount is the same.

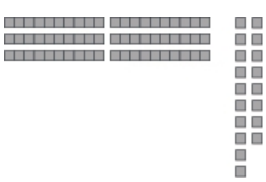
Here is a larger example using base-10 blocks and the Area Model of Multiplication.
78 divided by 3 can be thought of as computing the number that when multiplied by 3 equals 78.
Thought of as a rectangle, one side is 3 and the area is 78. What does the other side have to be?
One way to solve this problem is to get 78 as base-10 blocks, and using all of the blocks make a single rectangle where one of the sides is 3.
The other side will be the solution to the division problem.

We could get 78 units but that's a lot of work and doesn't allow us to take advantage of place value. Instead, start with 7 rods and 8 units, the normal way to represent 78.

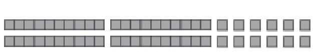


Make 3 equal rows using as many rods as possible. That's 2 rods per row. There is one rod left over along with the unused 8 units.

Exchange the remaining rod for 10 units. Combine that with the 8 units you already had giving you 18 units.



Now divide those 18 units equally over the 3 rows. Each row gets 6 units so all 18 are used and no base-10 blocks remain not in a row. Notice that each row has a value of 26, two rods and 6 units. 26 must be the answer to the original division problem. Check that 3 x 26 = 78.



This used the Distributive Property of Division because it broke up dividing 78 by 3 into dividing 70 by 3 and then 8 by 3. It was a little more complicated than just using the distributive property because the one remaining

rod had to converted into units so that 18 could be divided by 3.

When the divisor or the dividend has more than one digit, it is usually easier to use the $4\overline{)12}$ notation when working on paper. This process is called long division. Let’s work through the process by dividing 78 by 3. You should see how it directly follows what we just did with base-10 blocks.

Divide the first digit of the dividend, 7, by the divisor, 3.	
The divisor 3 can go into 7 two times since $2 \times 3 = 6$. Write the 2 above the 7 in the quotient.	$\begin{array}{r} 2 \\ 3\overline{)78} \end{array}$
Multiply the 2 in the quotient by 2 and write the product, 6, under the 7.	$\begin{array}{r} 2 \\ 3\overline{)78} \\ \underline{6} \end{array}$
Subtract that product from the first digit in the dividend. Subtract $7 - 6$. Write the difference, 1, under the first digit in the dividend.	$\begin{array}{r} 2 \\ 3\overline{)78} \\ \underline{6} \\ 1 \end{array}$
Bring down the next digit of the dividend. Bring down the 8.	$\begin{array}{r} 2 \\ 3\overline{)78} \\ \underline{6} \\ 18 \end{array}$
Divide 18 by the divisor, 3. The divisor 3 goes into 18 six times.	$\begin{array}{r} 26 \\ 3\overline{)78} \\ \underline{6} \\ 18 \end{array}$
Write 6 in the quotient above the 8.	
Multiply the 6 in the quotient by the divisor and write the product, 18, under the dividend. Subtract 18 from 18.	$\begin{array}{r} 26 \\ 3\overline{)78} \\ \underline{6} \\ 18 \\ \underline{18} \\ 0 \end{array}$

We would repeat the process until there are no more digits in the dividend to bring down. In this problem, there are no more digits to bring down, so the division is finished.
Equation:

$$\text{So } 78 \div 3 = 26.$$

Money Model
For practice, do this same problem with money. Start with 7 $10 bills and 8 $1 bills. Divide the $78 into three equal piles.
As you are dividing the money, try to follow the long division steps shown earlier.

Check Division with Multiplication
Check by multiplying the quotient times the divisor to get the dividend. Multiply 26 by 3 to make sure that product equals the dividend, 78.
Equation:

$$\begin{array}{r} \overset{1}{2}6 \\ \underline{\times 3} \\ 78\checkmark \end{array}$$

It does, so our answer is correct.

Note:
Divide whole numbers.

Divide the first digit of the dividend by the divisor.
If the divisor is larger than the first digit of the dividend, divide the first two digits of the dividend by the divisor, and so on.
Write the quotient above the dividend.
Multiply the quotient by the divisor and write the product under the dividend.
Subtract that product from the dividend.
Bring down the next digit of the dividend.
Repeat from Step 1 until there are no more digits in the dividend to bring down.
Check by multiplying the quotient times the divisor.

Example:
Exercise:

Problem: Divide $2{,}596 \div 4$. Check by multiplying:

Solution:
Solution

Let's rewrite the problem to set it up for long division.	$4\overline{)2596}$
Divide the first digit of the dividend, 2, by the divisor, 4.	$4\overline{)2596}$
Since 4 does not go into 2, we use the first two digits of the dividend and divide 25 by 4. The divisor 4 goes into 25 six times.	
We write the 6 in the quotient above the 5.	$\begin{array}{r} 6 \\ 4\overline{)2596} \end{array}$
Multiply the 6 in the quotient by the divisor 4 and write the product, 24, under the first two digits in the dividend.	$\begin{array}{r} 6 \\ 4\overline{)2596} \\ \underline{24} \end{array}$

Subtract that product from the first two digits in the dividend. Subtract $25 - 24$. Write the difference, 1, under the second digit in the dividend.	$\begin{array}{r} 6 \\ 4\overline{)2596} \\ \underline{24} \\ 1 \end{array}$
Now bring down the 9 and repeat these steps. There are 4 fours in 19. Write the 4 over the 9. Multiply the 4 by 4 and subtract this product from 19.	$\begin{array}{r} 64 \\ 4\overline{)2596} \\ \underline{24} \\ 19 \\ \underline{16} \\ 3 \end{array}$
Bring down the 6 and repeat these steps. There are 9 fours in 36. Write the 9 over the 6. Multiply the 9 by 4 and subtract this product from 36.	$\begin{array}{r} 649 \\ 4\overline{)2596} \\ \underline{24} \\ 19 \\ \underline{16} \\ 36 \\ \underline{36} \\ 0 \end{array}$
So $2{,}596 \div 4 = 649$.	
Check by multiplying. $\begin{array}{r} {}^{1\,3} \\ 649 \\ \underline{\times \quad 4} \\ 2{,}596 \checkmark \end{array}$	

It equals the dividend, so our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: $2{,}636 \div 4$

Solution:

659

Note:
Exercise:

Problem: Divide. Then check by multiplying: $2{,}716 \div 4$

Solution:

679

Example:
Exercise:

Problem: Divide 4,506 ÷ 6. Check by multiplying:

Solution:
Solution

Let's rewrite the problem to set it up for long division.	$6\overline{)4506}$
First we try to divide 6 into 4.	$6\overline{)4506}$
Since that won't work, we try 6 into 45. There are 7 sixes in 45. We write the 7 over the 5.	$\begin{array}{r} 7 \\ 6\overline{)4506} \end{array}$
Multiply the 7 by 6 and subtract this product from 45.	$\begin{array}{r} 7 \\ 6\overline{)4506} \\ 42 \\ \hline 3 \end{array}$
Now bring down the 0 and repeat these steps. There are 5 sixes in 30. Write the 5 over the 0. Multiply the 5 by 6 and subtract this product from 30.	$\begin{array}{r} 75 \\ 6\overline{)4506} \\ 42 \\ \hline 30 \\ 30 \\ \hline 0 \end{array}$
Now bring down the 6 and repeat these steps. There is 1 six in 6. Write the 1 over the 6. Multiply 1 by 6 and subtract this product from 6.	$\begin{array}{r} 751 \\ 6\overline{)4506} \\ 42 \\ \hline 30 \\ 30 \\ \hline 06 \\ 6 \\ \hline 0 \end{array}$
Check by multiplying. $\begin{array}{r} \overset{3}{7}51 \\ \times\quad 6 \\ \hline 4{,}506 \checkmark \end{array}$	

It equals the dividend, so our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: $4,305 \div 5$.

Solution:

861

Note:
Exercise:

Problem: Divide. Then check by multiplying: $3,906 \div 6$.

Solution:

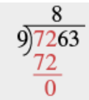
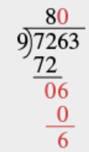
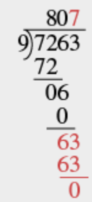
651

Example:
Exercise:

Problem: Divide $7,263 \div 9$. Check by multiplying.

Solution:
Solution

Let's rewrite the problem to set it up for long division.	$9\overline{)7263}$
First we try to divide 9 into 7.	$9\overline{)7263}$
Since that won't work, we try 9 into 72. There are 8 nines in 72. We write the 8 over the 2.	$\begin{array}{r} 8 \\ 9\overline{)7263} \end{array}$

Multiply the 8 by 9 and subtract this product from 72.	
Now bring down the 6 and repeat these steps. There are 0 nines in 6. Write the 0 over the 6. Multiply the 0 by 9 and subtract this product from 6.	
Now bring down the 3 and repeat these steps. There are 7 nines in 63. Write the 7 over the 3. Multiply the 7 by 9 and subtract this product from 63.	
Check by multiplying. $\begin{array}{r} \overset{6}{8}07 \\ \times \quad 9 \\ \hline 7{,}263 \end{array}$ ✓	

It equals the dividend, so our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: 4,928 ÷ 7.

Solution:

704

Note:
Exercise:

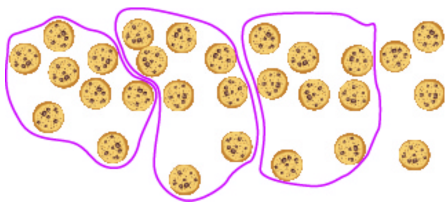
Problem: Divide. Then check by multiplying: 5,663 ÷ 7.

Solution:

809

So far all the division problems have worked out evenly. For example, if we had 24 cookies and wanted to make bags of 8 cookies, we would have 3 bags. But what if there were 28 cookies and we wanted to make bags of 8? Start with the 28 cookies as shown in [link].

Try to put the cookies in groups of eight as in [link].



There are 3 groups of eight cookies, and 4 cookies left over. We call the 4 cookies that are left over the remainder and show it by writing R4 next to the 3. (The R stands for remainder.)

To check this division we multiply 3 times 8 to get 24, and then add the remainder of 4.

Equation:

$$\begin{array}{r} 3 \\ \underline{\times 8} \\ 24 \\ \underline{+4} \\ 28 \end{array}$$

Example:

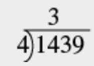
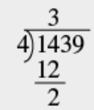
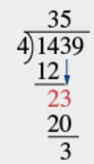
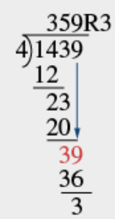
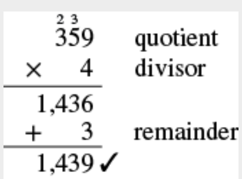
Exercise:

Problem: Divide $1,439 \div 4$. Check by multiplying.

Solution:

Solution

Let's rewrite the problem to set it up for long division.	$4\overline{)1439}$

First we try to divide 4 into 1. Since that won't work, we try 4 into 14. There are 3 fours in 14. We write the 3 over the 4.	
Multiply the 3 by 4 and subtract this product from 14.	
Now bring down the 3 and repeat these steps. There are 5 fours in 23. Write the 5 over the 3. Multiply the 5 by 4 and subtract this product from 23.	
Now bring down the 9 and repeat these steps. There are 9 fours in 39. Write the 9 over the 9. Multiply the 9 by 4 and subtract this product from 39. There are no more numbers to bring down, so we are done. The remainder is 3.	
Check by multiplying. 	

So $1,439 \div 4$ is 359 with a remainder of 3. Our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: $3,812 \div 8$.

Solution:

476 with a remainder of 4

Note:
Exercise:

Problem: Divide. Then check by multiplying: $4{,}319 \div 8$.

Solution:

539 with a remainder of 7

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation for division into words. Now we'll translate word phrases into math notation. Some of the words that indicate division are given in [link].

Operation	Word Phrase	Example	Expression
Division	divided by quotient of divided into	12 divided by 4 the quotient of 12 and 4 4 divided into 12	$12 \div 4$ $\frac{12}{4}$ $12/4$ $4\overline{)12}$

Example:
Exercise:

Problem: Translate and simplify: the quotient of 56 and 7.

Solution:
Solution

The word *quotient* tells us to divide.

	the quotient of 56 and 7
Translate.	$56 \div 7$
Divide.	8

We could just as correctly have translated *the quotient of* 56 *and* 7 using the notation

$7\overline{)56}$ or $\frac{56}{7}$.

Note:
Exercise:

Problem: Translate and simplify: the quotient of 96 and 8.

Solution:

$96 \div 8$; 12

Note:
Exercise:

Problem: Translate and simplify: the quotient of 52 and 4.

Solution:

$52 \div 4$; 13

Divide Whole Numbers in Applications

We will use the same strategy we used in previous sections to solve applications.

- Determine what we are asked to find.
- Write a phrase that gives the information to find it.
- Translate the phrase into math notation.
- Simplify to get the answer.
- Write a sentence to answer the question, using the appropriate units.

Example:
Exercise:

Problem:

Cecelia bought a 160-ounce box of oatmeal at the big box store. She wants to divide the 160 ounces of oatmeal into 8-ounce servings. She will put each serving into a plastic bag so she can take one bag to work each day. How many servings will she get from the big box?

Solution:
Solution

We are asked to find the how many servings she will get from the big box.

Write a phrase.	160 ounces divided by 8 ounces
Translate to math notation.	$160 \div 8$

Simplify by dividing.	20
Write a sentence to answer the question.	Cecelia will get 20 servings from the big box.

Note:
Exercise:

Problem:

Marcus is setting out animal crackers for snacks at the preschool. He wants to put 9 crackers in each cup. One box of animal crackers contains 135 crackers. How many cups can he fill from one box of crackers?

Solution:

Marcus can fill 15 cups.

Note:
Exercise:

Problem:

Andrea is making bows for the girls in her dance class to wear at the recital. Each bow takes 4 feet of ribbon, and 36 feet of ribbon are on one spool. How many bows can Andrea make from one spool of ribbon?

Solution:

Andrea can make 9 bows.

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Dividing Whole Numbers
- Dividing Whole Numbers No Remainder
- Dividing Whole Numbers With Remainder

Key Concepts

Operation	Notation	Expression	Read as	Result

Operation	Notation	Expression	Read as	Result
Division	$\div$ $\frac{a}{b}$ $b\overline{)a}$ a/b	$12 \div 4$ $\frac{12}{4}$ $4\overline{)12}$ $12/4$	Twelve divided by four	the quotient of 12 and 4

- **Division Properties of One**
    - Any number (except 0) divided by itself is one. $a \div a = 1$
    - Any number divided by one is the same number. $a \div 1 = a$
- **Division Properties of Zero**
    - Zero divided by any number is 0. $0 \div a = 0$
    - Dividing a number by zero is undefined. $a \div 0$ undefined
- **Divide whole numbers.**

  Divide the first digit of the dividend by the divisor. If the divisor is larger than the first digit of the dividend, divide the first two digits of the dividend by the divisor, and so on.
  Write the quotient above the dividend.
  Multiply the quotient by the divisor and write the product under the dividend.
  Subtract that product from the dividend.
  Bring down the next digit of the dividend.
  Repeat from Step 1 until there are no more digits in the dividend to bring down.
  Check by multiplying the quotient times the divisor.

Exercises

Practice Makes Perfect

Use Division Notation

In the following exercises, translate from math notation to words.

Exercise:

Problem: $54 \div 9$

Solution:

fifty-four divided by nine; the quotient of fifty-four and nine

Exercise:

Problem: $\frac{56}{7}$

Exercise:

Problem: $\frac{32}{8}$

Solution:

thirty-two divided by eight; the quotient of thirty-two and eight

Exercise:

Problem: $6\overline{)42}$

Exercise:

Problem: $48 \div 6$

Solution:

forty-eight divided by six; the quotient of forty-eight and six

Exercise:

Problem: $\frac{63}{9}$

Exercise:

Problem: $7\overline{)63}$

Solution:

sixty-three divided by seven; the quotient of sixty-three and seven

Exercise:

Problem: $72 \div 8$

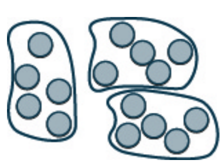
Model Division of Whole Numbers

In the following exercises, model the division.

Exercise:

Problem: $15 \div 5$

Solution:



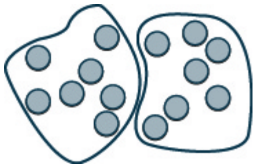
Exercise:

Problem: $10 \div 5$

Exercise:

Problem: $\frac{14}{7}$

Solution:



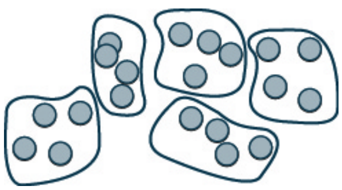
Exercise:

Problem: $\frac{18}{6}$

Exercise:

Problem: $4\overline{)20}$

Solution:



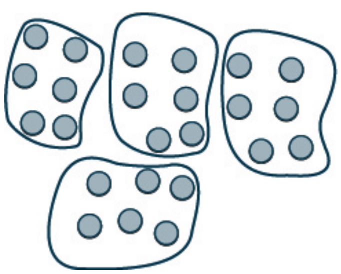
Exercise:

Problem: $3\overline{)15}$

Exercise:

Problem: $24 \div 6$

Solution:



Exercise:

Problem: $16 \div 4$

Divide Whole Numbers

In the following exercises, divide. Then check by multiplying.

Exercise:

Problem: $18 \div 2$

Solution:

9

Exercise:

Problem: $14 \div 2$

Exercise:

Problem: $\frac{27}{3}$

Solution:

9

Exercise:

Problem: $\frac{30}{3}$

Exercise:

Problem: $4\overline{)28}$

Solution:

7

Exercise:

Problem: $4\overline{)36}$

Exercise:

Problem: $\frac{45}{5}$

Solution:

9

Exercise:

Problem: $\frac{35}{5}$

Exercise:

Problem: $72/8$

Solution:

9

Exercise:

Problem: $8\overline{)64}$

Exercise:

Problem: $\frac{35}{7}$

Solution:

5

Exercise:

Problem: $42 \div 7$

Exercise:

Problem: $15\overline{)15}$

Solution:

1

Exercise:

Problem: $12\overline{)12}$

Exercise:

Problem: $43 \div 43$

Solution:

1

Exercise:

Problem: $37 \div 37$

Exercise:

Problem: $\frac{23}{1}$

Solution:

23

Exercise:

Problem: $\frac{29}{1}$

Exercise:

Problem: $19 \div 1$

Solution:

19

Exercise:

Problem: $17 \div 1$

Exercise:

Problem: $0 \div 4$

Solution:

0

Exercise:

Problem: $0 \div 8$

Exercise:

Problem: $\frac{5}{0}$

Solution:

undefined

Exercise:

Problem: $\frac{9}{0}$

Exercise:

Problem: $\frac{26}{0}$

Solution:

undefined

Exercise:

Problem: $\frac{32}{0}$

Exercise:

Problem: $12\overline{)0}$

Solution:

0

Exercise:

Problem: $16\overline{)0}$

Exercise:

Problem: $72 \div 3$

Solution:

24

Exercise:

Problem: $57 \div 3$

Exercise:

Problem: $\frac{96}{8}$

Solution:

12

Exercise:

Problem: $\frac{78}{6}$

Exercise:

Problem: $5\overline{)465}$

Solution:

93

Exercise:

Problem: $4\overline{)528}$

Exercise:

Problem: $924 \div 7$

Solution:

132

Exercise:

Problem: $861 \div 7$

Exercise:

Problem: $\frac{5{,}226}{6}$

Solution:

871

Exercise:

Problem: $\frac{3,776}{8}$

Exercise:

Problem: $4\overline{)31,324}$

Solution:

7,831

Exercise:

Problem: $5\overline{)46,855}$

Exercise:

Problem: $7,209 \div 3$

Solution:

2,403

Exercise:

Problem: $4,806 \div 3$

Exercise:

Problem: $5,406 \div 6$

Solution:

901

Exercise:

Problem: $3,208 \div 4$

Exercise:

Problem: $4\overline{)2,816}$

Solution:

704

Exercise:

Problem: $6\overline{)3,624}$

Exercise:

Problem: $\frac{91,881}{9}$

Solution:

10,209

Exercise:

Problem: $\frac{83,256}{8}$

Exercise:

Problem: $2,470 \div 7$

Solution:

352 R6

Exercise:

Problem: $3,741 \div 7$

Exercise:

Problem: $8\overline{)55,305}$

Solution:

6,913 R1

Exercise:

Problem: $9\overline{)51,492}$

Exercise:

Problem: $\frac{431,174}{5}$

Solution:

86,234 R4

Exercise:

Problem: $\frac{297,277}{4}$

Exercise:

Problem: $130,016 \div 3$

Solution:

43,338 R2

Exercise:

Problem: $105{,}609 \div 2$

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: $15\,(204)$

Solution:

3,060

Exercise:

Problem: $74 \cdot 391$

Exercise:

Problem: $256 - 184$

Solution:

72

Exercise:

Problem: $305 - 262$

Exercise:

Problem: $719 + 341$

Solution:

1,060

Exercise:

Problem: $647 + 528$

Exercise:

Problem: $2\overline{)875}$

Solution:

437R1

Exercise:

Problem: $1104 \div 3$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate and simplify.

Exercise:

Problem: the quotient of 45 and 5

Solution:

$45 \div 5$; 9

Exercise:

Problem: the quotient of 64 and 6

Exercise:

Problem: the quotient of 288 and 9

Solution:

$288 \div 9$; 32

Exercise:

Problem: the quotient of 256 and 8

Divide Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Trail mix Ric bought 64 ounces of trail mix. He wants to divide it into small bags, with 2 ounces of trail mix in each bag. How many bags can Ric fill?

Solution:

Ric can fill 32 bags.

Exercise:

Problem:

Crackers Evie bought a 42 ounce box of crackers. She wants to divide it into bags with 3 ounces of crackers in each bag. How many bags can Evie fill?

Exercise:

Problem:

Astronomy class There are 125 students in an astronomy class. The professor assigns them into groups of 5. How many groups of students are there?

Solution:

There are 25 groups.

Exercise:

Problem:

Flower shop Melissa's flower shop got a shipment of 152 roses. She wants to make bouquets of 8 roses each. How many bouquets can Melissa make?

Exercise:

Problem:

Baking One roll of plastic wrap is 48 feet long. Marta uses 3 feet of plastic wrap to wrap each cake she bakes. How many cakes can she wrap from one roll?

Solution:

Marta can wrap 16 cakes from 1 roll.

Exercise:

Problem:

Dental floss One package of dental floss is 54 feet long. Brian uses 2 feet of dental floss every day. How many days will one package of dental floss last Brian?

Mixed Practice

In the following exercises, solve.

Exercise:

Problem:

Miles per gallon Susana's hybrid car gets 45 miles per gallon. Her son's truck gets 17 miles per gallon. What is the difference in miles per gallon between Susana's car and her son's truck?

Solution:

The difference is 28 miles per gallon.

Exercise:

Problem:

Distance Mayra lives 53 miles from her mother's house and 71 miles from her mother-in-law's house. How much farther is Mayra from her mother-in-law's house than from her mother's house?

Exercise:

Problem:

Field trip The 45 students in a Geology class will go on a field trip, using the college's vans. Each van can hold 9 students. How many vans will they need for the field trip?

Solution:

They will need 5 vans for the field trip

Exercise:

Problem:

Potting soil Aki bought a 128 ounce bag of potting soil. How many 4 ounce pots can he fill from the bag?

Exercise:

Problem:

Hiking Bill hiked 8 miles on the first day of his backpacking trip, 14 miles the second day, 11 miles the third day, and 17 miles the fourth day. What is the total number of miles Bill hiked?

Solution:

Bill hiked 50 miles

Exercise:

Problem:

Reading Last night Emily read 6 pages in her Business textbook, 26 pages in her History text, 15 pages in her Psychology text, and 9 pages in her math text. What is the total number of pages Emily read?

Exercise:

Problem:

Patients LaVonne treats 12 patients each day in her dental office. Last week she worked 4 days. How many patients did she treat last week?

Solution:

LaVonne treated 48 patients last week.

Exercise:

Problem:

Scouts There are 14 boys in Dave's scout troop. At summer camp, each boy earned 5 merit badges. What was the total number of merit badges earned by Dave's scout troop at summer camp?

Writing Exercises

Exercise:

Problem: Explain how you use the multiplication facts to help with division.

Solution:

Answers may vary. Using multiplication facts can help you check your answers once you've finished division.

Exercise:

Problem:

Oswaldo divided 300 by 8 and said his answer was 37 with a remainder of 4. How can you check to make sure he is correct?

Everyday Math

Exercise:

Problem:

Milk Together Jenna and Dee drink a gallon of milk every 5 days. How many gallons do they drink every 365 days?

Solution:

365 days divided by 5 equals 73. Jenna and Dee drink 73 gallons.

Exercise:

Problem:

Cat food One can of cat food feeds Lara's cat for 3 days. How many cans of cat food does Lara need for 365 days?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use division notation.			
model division of whole numbers.			
divide whole numbers.			
translate word phrases to algebraic expressions.			
divide whole numbers in applications.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Review Exercises

[Introduction to Whole Numbers]

Identify Counting Numbers and Whole Numbers

In the following exercises, determine which of the following are (a) counting numbers (b) whole numbers.

Exercise:

Problem: $0, 2, 99$

Solution:

ⓐ 2, 99
ⓑ 0, 2, 99

Exercise:

Problem: $0, 3, 25$

Exercise:

Problem: $0, 4, 90$

Solution:

ⓐ 4, 90
ⓑ 0, 4, 90

Exercise:

Problem: $0, 1, 75$

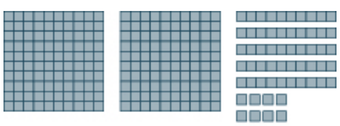
Model Whole Numbers

In the following exercises, model each number using base-10 blocks and then show its value using place value notation.

Exercise:

Problem: 258

Solution:



Place Value	Digit	Total Value
hundreds	2	200
tens	5	50
ones	8	8
		258

Exercise:

Problem: 104

Identify the Place Value of a Digit

In the following exercises, find the place value of the given digits.

Exercise:

Problem: 472,981

ⓐ 8
ⓑ 4
ⓒ 1
ⓓ 7
ⓔ 2

Solution:

ⓐ tens
ⓑ hundred thousands
ⓒ ones
ⓓ thousands
ⓔ ten thousands

Exercise:

Problem: 12,403,295

ⓐ 4
ⓑ 0
ⓒ 1
ⓓ 9
ⓔ 3

Use Place Value to Name Whole Numbers

In the following exercises, name each number in words.

Exercise:

Problem: 5,280

Solution:

Five thousand two hundred eighty

Exercise:

Problem: 204,614

Exercise:

Problem: 5,012,582

Solution:

Five million twelve thousand five hundred eighty-two

Exercise:

Problem: 31,640,976

Use Place Value to Write Whole Numbers

In the following exercises, write as a whole number using digits.

Exercise:

Problem: six hundred two

Exercise:

Problem: fifteen thousand, two hundred fifty-three

Solution:

15,253

Exercise:

Problem: three hundred forty million, nine hundred twelve thousand, sixty-one

---

Solution:

340,912,061

Exercise:

Problem: two billion, four hundred ninety-two million, seven hundred eleven thousand, two

Round Whole Numbers

In the following exercises, round to the nearest ten.

Exercise:

Problem: 412

Solution:

410

Exercise:

Problem: 648

Exercise:

Problem: 3,556

Solution:

3,560

Exercise:

Problem: 2,734

In the following exercises, round to the nearest hundred.

Exercise:

Problem: 38,975

Solution:

39,000

Exercise:

Problem: $26{,}849$

Exercise:

Problem: $81{,}486$

Solution:

$81{,}500$

Exercise:

Problem: $75{,}992$

Add Whole Numbers

Use Addition Notation

In the following exercises, translate the following from math notation to words.

Exercise:

Problem: $4+3$

Solution:

four plus three; the sum of four and three

Exercise:

Problem: $25+18$

Exercise:

Problem: $571+629$

Solution:

five hundred seventy-one plus six hundred twenty-nine; the sum of five hundred seventy-one and six hundred twenty-nine

Exercise:

Problem: $10{,}085+3{,}492$

Model Addition of Whole Numbers

In the following exercises, model the addition.

Exercise:

Problem: $6+7$

Solution:

Exercise:

Problem: $38 + 14$

Add Whole Numbers

In the following exercises, fill in the missing values in each chart.

Exercise:

Problem:

+	0	1	2	3	4	5	6	7	8	9
0	0	1		3	4		6	7		9
1	1	2	3	4			7	8	9	
2		3	4	5	6	7	8		10	11
3	3		5		7	8		10		12
4	4	5			8	9			12	
5	5		7	8			11		13	
6	6	7	8		10			13		15
7			9			12	13		15	16
8	8	9		11			14		16	
9	9	10	11		13	14			17	

Solution:

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Exercise:

Problem:

+	3	4	5	6	7	8	9
6							
7							
8							
9							

In the following exercises, add.

Exercise:

Problem: ⓐ $0 + 19$ ⓑ $19 + 0$

Solution:

ⓐ 19
ⓑ 19

Exercise:

Problem: ⓐ $0 + 480$ ⓑ $480 + 0$

Exercise:

Problem: ⓐ $7 + 6$ ⓑ $6 + 7$

Solution:

ⓐ 13
ⓑ 13

Exercise:

Problem: ⓐ $23 + 18$ ⓑ $18 + 23$

Exercise:

Problem: $44 + 35$

Solution:

82

Exercise:

Problem: $63 + 29$

Exercise:

Problem: $96 + 58$

Solution:

154

Exercise:

Problem: $375 + 591$

Exercise:

Problem: $7{,}281 + 12{,}546$

Solution:

19,827

Exercise:

Problem: $5{,}280 + 16{,}324 + 9{,}731$

Translate Word Phrases to Math Notation

In the following exercises, translate each phrase into math notation and then simplify.

Exercise:

Problem: the sum of 30 and 12

Solution:

30 + 12; 42

Exercise:

Problem: 11 increased by 8

Exercise:

Problem: 25 more than 39

Solution:

39 + 25; 64

Exercise:

Problem: total of 15 and 50

Add Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Shopping for an interview Nathan bought a new shirt, tie, and slacks to wear to a job interview. The shirt cost \$24, the tie cost \$14, and the slacks cost \$38. What was Nathan’s total cost?

Solution:

$76

Exercise:

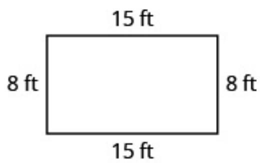
Problem:

Running Jackson ran 4 miles on Monday, 12 miles on Tuesday, 1 mile on Wednesday, 8 miles on Thursday, and 5 miles on Friday. What was the total number of miles Jackson ran?

In the following exercises, find the perimeter of each figure.

Exercise:

Problem:

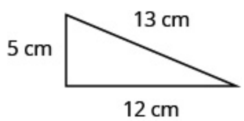


Solution:

46 feet

Exercise:

Problem:



Subtract Whole Numbers

Use Subtraction Notation

In the following exercises, translate the following from math notation to words.

Exercise:

Problem: $14 - 5$

Solution:

fourteen minus five; the difference of fourteen and five

Exercise:

Problem: $40 - 15$

Exercise:

Problem: $351 - 249$

Solution:

three hundred fifty-one minus two hundred forty-nine; the difference between three hundred fifty-one and two hundred forty-nine

Exercise:

Problem: $5{,}724 - 2{,}918$

Model Subtraction of Whole Numbers

In the following exercises, model the subtraction.

Exercise:

Problem: $18 - 4$

Solution:

Exercise:

Problem: $41 - 29$

Subtract Whole Numbers

In the following exercises, subtract and then check by adding.

Exercise:

Problem: $8 - 5$

Solution:

3

Exercise:

Problem: $12 - 7$

Exercise:

Problem: $23 - 9$

Solution:

14

Exercise:

Problem: $46 - 21$

Exercise:

Problem: $82 - 59$

Solution:

23

Exercise:

Problem: $110 - 87$

Exercise:

Problem: $539 - 217$

Solution:

322

Exercise:

Problem: $415 - 296$

Exercise:

Problem: $1{,}020 - 640$

Solution:

380

Exercise:

Problem: $8{,}355 - 3{,}947$

Exercise:

Problem: $10{,}000 - 15$

Solution:

9,985

Exercise:

Problem: $54{,}925 - 35{,}647$

Translate Word Phrases to Math Notation

In the following exercises, translate and simplify.

Exercise:

Problem: the difference of nineteen and thirteen

Solution:

19 − 13; 6

Exercise:

Problem: subtract sixty-five from one hundred

Exercise:

Problem: seventy-four decreased by eight

Solution:

$74 - 8; 66$

Exercise:

Problem: twenty-three less than forty-one

Subtract Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature The high temperature in Peoria one day was 86 degrees Fahrenheit and the low temperature was 28 degrees Fahrenheit. What was the difference between the high and low temperatures?

Solution:

58 degrees Fahrenheit

Exercise:

Problem:

Savings Lynn wants to go on a cruise that costs $\$2,485$. She has $\$948$ in her vacation savings account. How much more does she need to save in order to pay for the cruise?

Multiply Whole Numbers

Use Multiplication Notation

In the following exercises, translate from math notation to words.

Exercise:

Problem: 8×5

Solution:

eight times five the product of eight and five

Exercise:

Problem: $6 \cdot 14$

Exercise:

Problem: $(10)(95)$

Solution:

ten times ninety-five; the product of ten and ninety-five

Exercise:

Problem: $54(7)$

Model Multiplication of Whole Numbers

In the following exercises, model the multiplication.

Exercise:

Problem: 2×4

Solution:

Exercise:

Problem: 3×8

Multiply Whole Numbers

In the following exercises, fill in the missing values in each chart.

Exercise:

Problem:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0		0		0	0
1	0	1	2		4	5	6	7		9
2	0		4		8	10		14	16	
3		3		9			18		24	
4	0	4		12			24			36
5	0	5	10		20		30	35	40	45
6			12	18			36	42		54
7	0	7		21		35			56	63
8	0	8	16		32		48		64	
9			18	27	36			63	72	

Solution:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Exercise:

Problem:

×	3	4	5	6	7	8	9
6							
7							
8							
9							

In the following exercises, multiply.

Exercise:

Problem: $0 \cdot 14$

Solution:

0

Exercise:

Problem: $(256)0$

Exercise:

Problem: $1 \cdot 99$

Solution:

99

Exercise:

Problem: $(4{,}789)1$

Exercise:

Problem: ⓐ $7 \cdot 4$ ⓑ $4 \cdot 7$

Solution:

ⓐ 28
ⓑ 28

Exercise:

Problem: $(25)(6)$

Exercise:

Problem: $9{,}261 \times 3$

Solution:

27,783

Exercise:

Problem: $48 \cdot 6$

Exercise:

Problem: $64 \cdot 10$

Solution:

640

Exercise:

Problem: $1{,}000(22)$

Exercise:

Problem: 162×4

Solution:

648

Exercise:

Problem: $(6)(943)$

Exercise:

Problem: $3{,}624 \times 7$

Solution:

25,368

Exercise:

Problem: $10{,}538 \cdot 2$

Translate Word Phrases to Math Notation

In the following exercises, translate and simplify.
Exercise:

Problem: the product of 5 and 28

Solution:

5(28); 140

Exercise:

Problem: ninety-four times three

Exercise:

Problem: twice 575

Solution:

2(575); 1,150

Exercise:

Problem: nine times two hundred sixty-four

Multiply Whole Numbers in Applications

In the following exercises, solve.
Exercise:

Problem:

Gardening Geniece bought 8 packs of marigolds to plant in her yard. Each pack has 6 flowers. How many marigolds did Geniece buy?

Solution:

48 marigolds

Exercise:

Problem:

Cooking Ratika is making rice for a dinner party. The number of cups of water is twice the number of cups of rice. If Ratika plans to use 4 cups of rice, how many cups of water does she need?

Exercise:

Problem:

Multiplex There are seven theaters at the multiplex and each theater has 350 seats. What is the total number of seats at the multiplex?

Solution:

2,450 seats

Exercise:

Problem:

Roofing Lewis needs to put new shingles on his roof. The roof is a rectangle, 12 yards by 8 yards. What is the area of the roof?

Divide Whole Numbers

Use Division Notation

Translate from math notation to words.

Exercise:

Problem: $54 \div 9$

Solution:

fifty-four divided by nine; the quotient of fifty-four and nine

Exercise:

Problem: $42/7$

Exercise:

Problem: $\frac{72}{8}$

Solution:

seventy-two divided by eight; the quotient of seventy-two and eight

Exercise:

Problem: $6\overline{)48}$

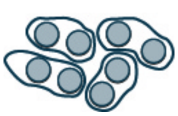
Model Division of Whole Numbers

In the following exercises, model.

Exercise:

Problem: $8 \div 2$

Solution:



Exercise:

Problem: $3\overline{)12}$

Divide Whole Numbers

In the following exercises, divide. Then check by multiplying.

Exercise:

Problem: $14 \div 2$

Solution:

7

Exercise:

Problem: $\frac{32}{8}$

Exercise:

Problem: $52 \div 4$

Solution:

13

Exercise:

Problem: $26\overline{)26}$

Exercise:

Problem: $\frac{97}{1}$

Solution:

97

Exercise:

Problem: $0 \div 52$

Exercise:

Problem: $100 \div 0$

Solution:

undefined

Exercise:

Problem: $\frac{355}{5}$

Exercise:

Problem: $3828 \div 6$

Solution:

638

Exercise:

Problem: $6\overline{)2,562}$

Exercise:

Problem: $\frac{7505}{7}$

Solution:

1,072 R1

Exercise:

Problem: $5,006 \div 4$

Translate Word Phrases to Math Notation

In the following exercises, translate and simplify.

Exercise:

Problem: the quotient of 64 and 2

Solution:

64 ÷ 2; 32

Exercise:

Problem: the quotient of 572 and 5

Divide Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Ribbon One spool of ribbon is 27 feet. Lizbeth uses 3 feet of ribbon for each gift basket that she wraps. How many gift baskets can Lizbeth wrap from one spool of ribbon?

Solution:

9 baskets

Exercise:

Problem:

Juice One carton of fruit juice is 128 ounces. How many 4 ounce cups can Shayla fill from one carton of juice?

Practice Test

Exercise:

Problem: Determine which of the following numbers are

ⓐ counting numbers
ⓑ whole numbers.

$0, 4, 87$

Solution:

ⓐ 4, 87
ⓑ 0, 4, 8

Exercise:

Problem: Find the place value of the given digits in the number $549{,}362$.

ⓐ 9
ⓑ 6
ⓒ 2
ⓓ 5

Exercise:

Problem: Write each number as a whole number using digits.

ⓐ six hundred thirteen
ⓑ fifty-five thousand two hundred eight

Solution:

ⓐ 613
ⓑ 55,208

Exercise:

Problem: Round $25{,}849$ to the nearest hundred.

Simplify.
Exercise:

Problem: $45 + 23$

Solution:

68

Exercise:

Problem: $65 - 42$

Exercise:

Problem: $85 \div 5$

Solution:

17

Exercise:

Problem: $1{,}000 \ \times \ 8$

Exercise:

Problem: $90 - 58$

Solution:

32

Exercise:

Problem: $73 + 89$

Exercise:

Problem: $(0)(12{,}675)$

Solution:

0

Exercise:

Problem: $634 + 255$

Exercise:

Problem: $\frac{0}{9}$

Solution:

0

Exercise:

Problem: $8\overline{)128}$

Exercise:

Problem: $145 - 79$

Solution:

66

Exercise:

Problem: $299 + 836$

Exercise:

Problem: $7 \cdot 475$

Solution:

$3{,}325$

Exercise:

Problem: $8{,}528 + 704$

Exercise:

Problem: $35(7)$

Solution:

245

Exercise:

Problem: $\frac{26}{0}$

Exercise:

Problem: $733 - 291$

Solution:

442

Exercise:

Problem: $4{,}916 - 1{,}538$

Exercise:

Problem: $495 \div 9$

Solution:

55

Exercise:

Problem: 7×983

Translate each phrase to math notation and then simplify.

Exercise:

Problem: The sum of 16 and 58

Solution:

16 + 58; 74

Exercise:

Problem: The product of 9 and 15

Exercise:

Problem: The difference of 32 and 18

Solution:

32 − 18; 14

Exercise:

Problem: The quotient of 259 and 7

Exercise:

Problem: Twice 524

Solution:

2(524); 1,048

Exercise:

Problem: 29 more than 32

Exercise:

Problem: 50 less than 300

Solution:

300 − 50; 250

In the following exercises, solve.

Exercise:

Problem:

LaVelle buys a jumbo bag of 84 candies to make favor bags for her son's party. If she wants to make 7 bags, how many candies should she put in each bag?

Exercise:

Problem:

Last month, Stan's take-home pay was $\$3,816$ and his expenses were $\$3,472$. How much of his take-home pay did Stan have left after he paid his expenses?

Solution:

Stan had $344 left.

Exercise:

Problem:

Each grade at Greenville School has 220 children enrolled. The school has 4 classes. How many children are enrolled at Greenville School?

Exercise:

Problem:

Clayton walked 12 blocks to his mother's house, 6 blocks to the gym, and 9 blocks to the grocery store before walking the last 3 blocks home. What was the total number of blocks that Clayton walked?

Solution:

Clayton walked 30 blocks.

Glossary

dividend
: When dividing two numbers, the dividend is the number being divided.

divisor
: When dividing two numbers, the divisor is the number dividing the dividend.

quotient
: The quotient is the result of dividing two numbers.

# Introduction to Fractions
class="introduction"

Bakers combine ingredients to make delicious breads and pastries. (credit: Agustín Ruiz, Flickr)

Often in life, whole amounts are not exactly what we need. A baker must use a little more than a cup of milk or part of a teaspoon of sugar. Similarly a carpenter might need less than a foot of wood and a painter might use part of a gallon of paint. In this chapter, we will learn about numbers that describe parts of a whole. These numbers, called fractions, are very useful both in algebra and in everyday life. You will discover that you are already familiar with many examples of fractions!

Visualize Fractions Beginning Level
By the end of this section, you will be able to:

- Understand the meaning of fractions
- Model improper fractions and mixed numbers

Note:
Before you get started, take this readiness quiz.

1. Divide $1{,}439 \div 4$. If you missed this problem, review [link].

Understand the Meaning of Fractions

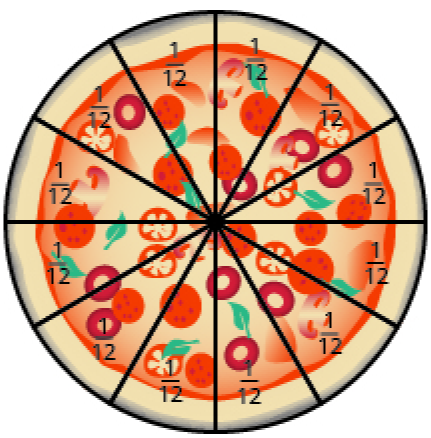
Andy and Bobby love pizza. On Monday night, they share a pizza equally. How much of the pizza does each one get? Are you thinking that each boy gets half of the pizza? That's right. There is one whole pizza, evenly divided into two parts, so each boy gets one of the two equal parts.

In math, we write $\frac{1}{2}$ to mean one out of two parts.

On Tuesday, Andy and Bobby share a pizza with their parents, Fred and Christy, with each person getting an equal amount of the whole pizza. How much of the pizza does each person get? There is one whole pizza, divided

evenly into four equal parts. Each person has one of the four equal parts, so each has $\frac{1}{4}$ of the pizza.

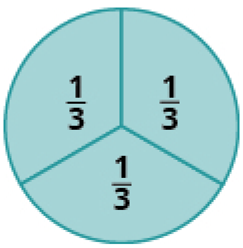
On Wednesday, the family invites some friends over for a pizza dinner. There are a total of 12 people. If they share the pizza equally, each person would get $\frac{1}{12}$ of the pizza.



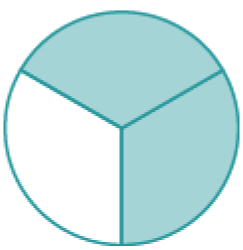
Note:
Fractions
A fraction is written $\frac{a}{b}$, where a and b are integers and $b \neq 0$. In a fraction, a is called the numerator and b is called the denominator.

A fraction is a way to represent parts of a whole. The denominator b represents the number of equal parts the whole has been divided into, and the numerator a represents how many parts are included. The denominator, b, cannot equal zero because division by zero is undefined.

In [link], the circle has been divided into three parts of equal size. Each part represents $\frac{1}{3}$ of the circle. This type of model is called a fraction circle. Other shapes, such as rectangles, can also be used to model fractions.



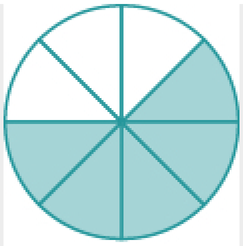
What does the fraction $\frac{2}{3}$ represent? The fraction $\frac{2}{3}$ means two of three equal parts.



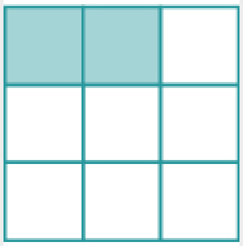
Example:
Exercise:

Problem:

Name the fraction of the shape that is shaded in each of the figures.



(a)



(b)

Solution:
Solution

We need to ask two questions. First, how many equal parts are there? This will be the denominator. Second, of these equal parts, how many are shaded? This will be the numerator.

ⓐ

How many equal parts are there?	There are eight equal parts.
How many are shaded?	Five parts are shaded.

Five out of eight parts are shaded. Therefore, the fraction of the circle that is shaded is $\frac{5}{8}$.

ⓑ

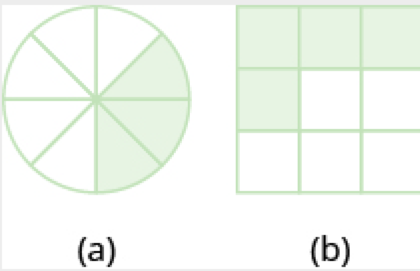
How many equal parts are there?	There are nine equal parts.
How many are shaded?	Two parts are shaded.

Two out of nine parts are shaded. Therefore, the fraction of the square that is shaded is $\frac{2}{9}$.

Note:
Exercise:

Problem:

Name the fraction of the shape that is shaded in each figure:



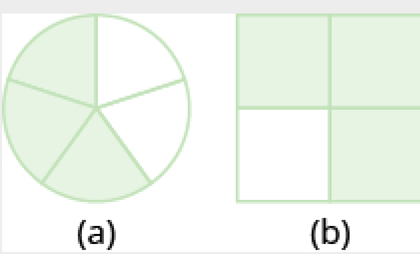
Solution:

ⓐ $\frac{3}{8}$
ⓑ $\frac{4}{9}$

Note:
Exercise:

Problem:

Name the fraction of the shape that is shaded in each figure:



Solution:

ⓐ $\frac{3}{5}$
ⓑ $\frac{3}{4}$

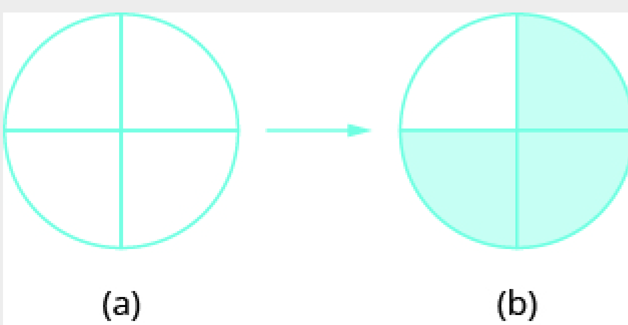
Example:
Exercise:

Problem: Shade $\frac{3}{4}$ of the circle.

Solution:
Solution

The denominator is 4, so we divide the circle into four equal parts ⓐ.

The numerator is 3, so we shade three of the four parts ⓑ.

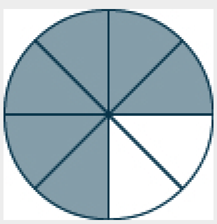


$\frac{3}{4}$ of the circle is shaded.

Note:
Exercise:

Problem: Shade $\frac{6}{8}$ of the circle.

Solution:

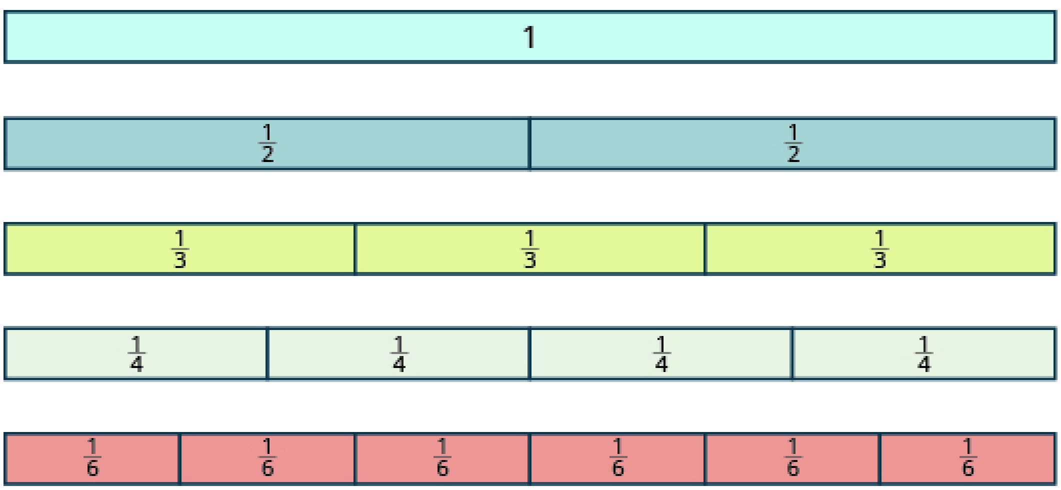


Note:
Exercise:

Problem: Shade $\frac{2}{5}$ of the rectangle.

Solution:

In [link] and [link], we used circles and rectangles to model fractions. Fractions can also be modeled as manipulatives called fraction towers, as shown in [link]. Here, the whole is modeled as one long, undivided rectangular tower. Beneath it are towers of equal length divided into different numbers of equally sized parts.



We’ll be using fraction towers to discover some basic facts about fractions. Refer to [link] to answer the following questions:

How many $\frac{1}{2}$ towers does it take to make one whole tower?	It takes two halves to make a whole, so two out of two is $\frac{2}{2} = 1$.
How many $\frac{1}{3}$ towers does it take to make one whole tower?	It takes three thirds, so three out of three is $\frac{3}{3} = 1$.
How many $\frac{1}{4}$ towers does it take to make one whole tower?	It takes four fourths, so four out of four is $\frac{4}{4} = 1$.
How many $\frac{1}{6}$ towers does it take to make one whole tower?	It takes six sixths, so six out of six is $\frac{6}{6} = 1$.
What if the whole were divided into 24 equal parts? (We have not shown fraction towers to represent this, but try to visualize it in your mind.) How many $\frac{1}{24}$ towers does it take to make one whole tower?	It takes 24 twenty-fourths, so $\frac{24}{24} = 1$.

It takes 24 twenty-fourths, so $\frac{24}{24} = 1$.

This leads us to the *Property of One*.

Note:
Property of One
Any number, except zero, divided by itself is one.
Equation:

$$\frac{a}{a} = 1 \qquad (a \neq 0)$$

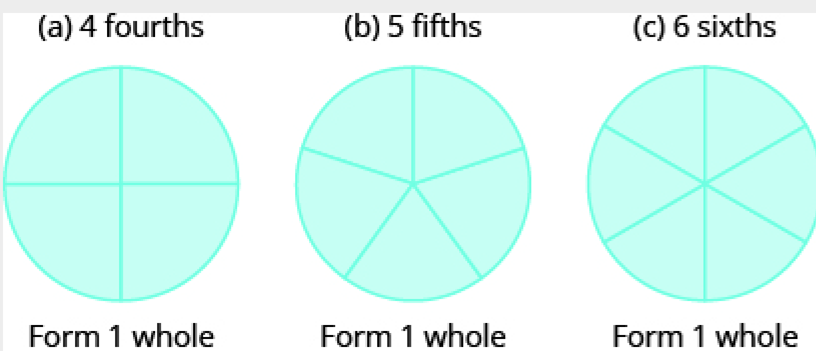
Example:
Exercise:

Problem:

Use fraction circles to make wholes using the following pieces:

ⓐ 4 fourths
ⓑ 5 fifths
ⓒ 6 sixths

Solution:
Solution



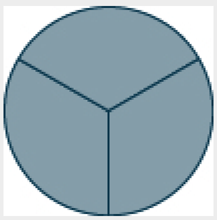
Note:

Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 3 thirds.

Solution:

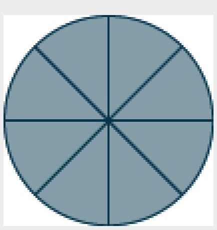


Note:
Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 8 eighths.

Solution:



What if we have more fraction pieces than we need for 1 whole? We'll look at this in the next example.

Example:
Exercise:

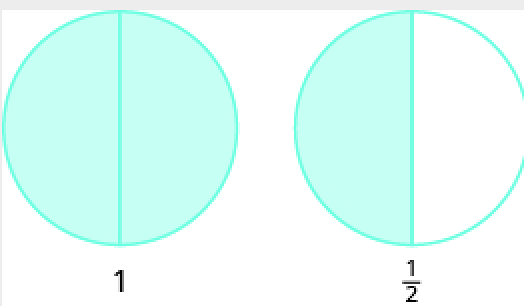
Problem:

Use fraction circles to make wholes using the following pieces:

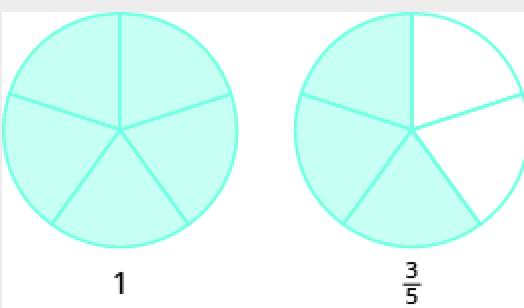
ⓐ 3 halves
ⓑ 8 fifths
ⓒ 7 thirds

Solution:
Solution

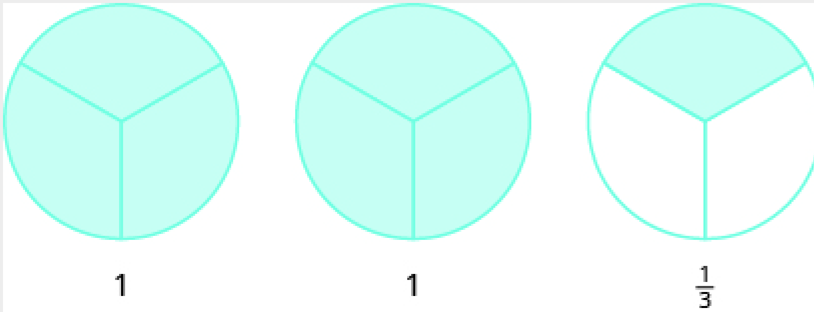
ⓐ3 halves make 1 whole with 1 half left over.



ⓑ8 fifths make 1 whole with 3 fifths left over.



ⓒ7 thirds make 2 wholes with 1 third left over.

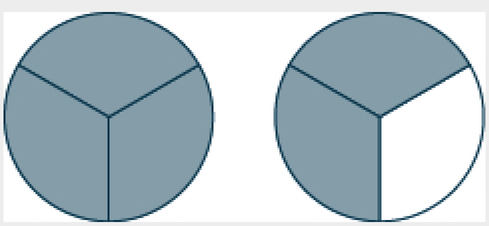


Note:
Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 5 thirds.

Solution:

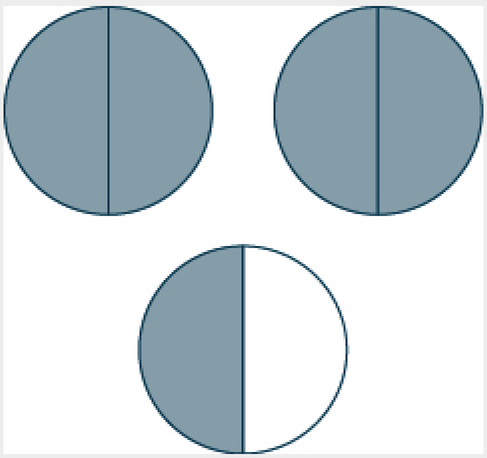


Note:
Exercise:

Problem:

Use fraction circles to make wholes with the following pieces: 5 halves.

Solution:



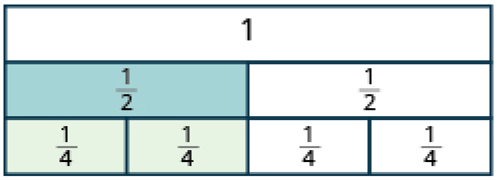
Model Equivalent Fractions

Let's think about Andy and Bobby and their favorite food again. If Andy eats $\frac{1}{2}$ of a pizza and Bobby eats $\frac{2}{4}$ of the pizza, have they eaten the same amount of pizza? In other words, does $\frac{1}{2} = \frac{2}{4}$? We can use fraction towers to find out whether Andy and Bobby have eaten *equivalent* parts of the pizza.

Note:
Equivalent Fractions
Equivalent fractions are fractions that have the same value.

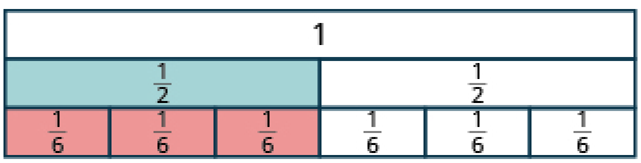
Fraction towers serve as a useful model of equivalent fractions. You may want to use fraction towers to do the following activity. Or you might make a copy of [link] and extend it to include eighths, tenths, and twelfths.

Start with a $\frac{1}{2}$ tower. How many fourths equal one-half? How many of the $\frac{1}{4}$ towers is exactly the same length as the $\frac{1}{2}$ tower?



Since two $\frac{1}{4}$ towers is the same as the $\frac{1}{2}$ tower, we see that $\frac{2}{4}$ is the same as $\frac{1}{2}$, or $\frac{2}{4} = \frac{1}{2}$.

How many of the $\frac{1}{6}$ towers is the same as the $\frac{1}{2}$ tower?



Since three $\frac{1}{6}$ towers is the same as the $\frac{1}{2}$ tower, we see that $\frac{3}{6}$ is the same as $\frac{1}{2}$.

So, $\frac{3}{6} = \frac{1}{2}$. The fractions are equivalent fractions.

Example:
Exercise:

Problem:

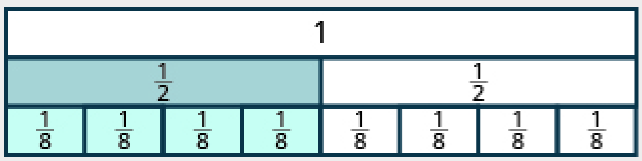
Use fraction towers to find equivalent fractions. Show your result with a figure.

ⓐ How many eighths equal one-half?
ⓑ How many tenths equal one-half?

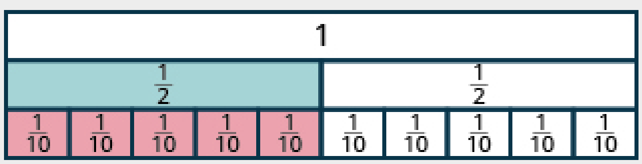
ⓒ How many twelfths equal one-half?

Solution:
Solution

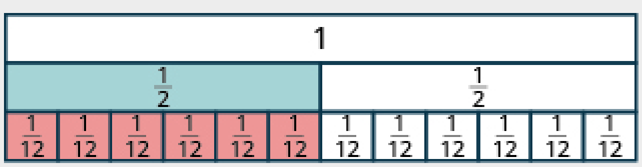
ⓐ It takes four $\frac{1}{8}$ towers to be exactly the same as the $\frac{1}{2}$ tower, so $\frac{4}{8} = \frac{1}{2}$.



ⓑ It takes five $\frac{1}{10}$ towers to be exactly the same as the $\frac{1}{2}$ tower, so $\frac{5}{10} = \frac{1}{2}$.



ⓒ It takes six $\frac{1}{12}$ towers to be exactly the same as the $\frac{1}{2}$ tower, so $\frac{6}{12} = \frac{1}{2}$.



Suppose you had towers marked $\frac{1}{20}$. How many of them would it take to equal $\frac{1}{2}$? Are you thinking ten towers? If you are, you're right, because $\frac{10}{20} = \frac{1}{2}$.

We have shown that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, and $\frac{10}{20}$ are all equivalent fractions.

Note:
Exercise:

Problem:

Use fraction towers to find equivalent fractions: How many eighths equal one-fourth?

Solution:

2

Note:
Exercise:

Problem:

Use fraction towers to find equivalent fractions: How many twelfths equal one-fourth?

Solution:

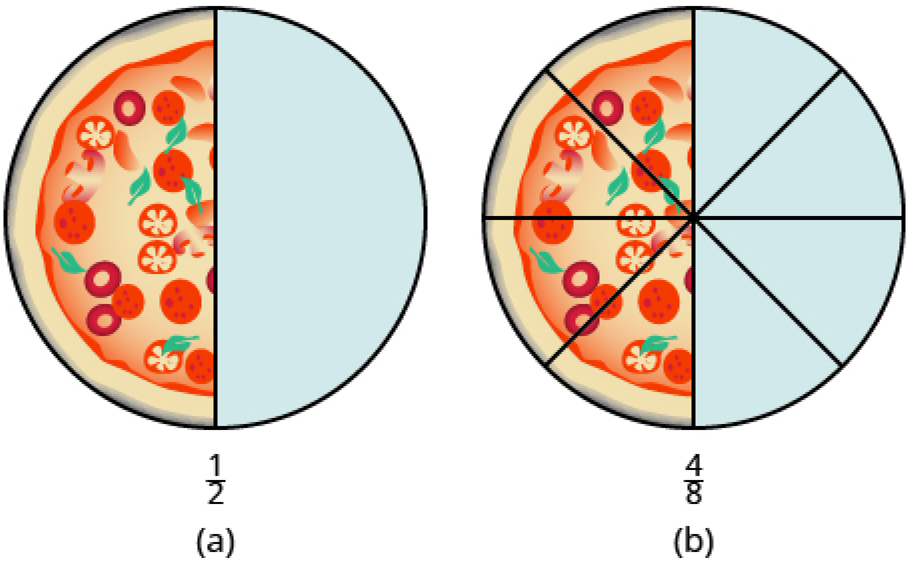
3

Find Equivalent Fractions

We used fraction towers to show that there are many fractions equivalent to $\frac{1}{2}$. For example, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are all equivalent to $\frac{1}{2}$. When we lined up the fraction towers, it took four of the $\frac{1}{8}$ towers to make the same length as a $\frac{1}{2}$ tower. This showed that $\frac{4}{8} = \frac{1}{2}$. See [link].

We can show this with pizzas, too. [link](a) shows a single pizza, cut into two equal pieces with $\frac{1}{2}$ shaded. [link](b) shows a second pizza of the same

size, cut into eight pieces with $\frac{4}{8}$ shaded.



This is another way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$.

How can we use mathematics to change $\frac{1}{2}$ into $\frac{4}{8}$? How could you take a pizza that is cut into two pieces and cut it into eight pieces? You could cut each of the two larger pieces into four smaller pieces! The whole pizza would then be cut into eight pieces instead of just two. Mathematically, what we've described could be written as:

$$\frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$$

These models lead to the Equivalent Fractions Property, which states that if we multiply the numerator and denominator of a fraction by the same number, the value of the fraction does not change.

Note:
Equivalent Fractions Property
If a, b, and c are numbers where $b \neq 0$ and $c \neq 0$, then
Equation:

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

When working with fractions, it is often necessary to express the same fraction in different forms. To find equivalent forms of a fraction, we can use the Equivalent Fractions Property. For example, consider the fraction one-half.

$$\frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \quad \text{so} \quad \frac{1}{2} = \frac{3}{6}$$

$$\frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \quad \text{so} \quad \frac{1}{2} = \frac{2}{4}$$

$$\frac{1 \cdot 10}{2 \cdot 10} = \frac{10}{20} \quad \text{so} \quad \frac{1}{2} = \frac{10}{20}$$

So, we say that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{10}{20}$ are equivalent fractions.

Example:
Exercise:

Problem: Find three fractions equivalent to $\frac{2}{5}$.

Solution:
Solution

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number (but not zero). Let us multiply them by 2, 3, and 5.

$$\frac{2 \cdot 2}{5 \cdot 2} = \frac{4}{10} \qquad \frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15} \qquad \frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$$

So, $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Note:
Exercise:

Problem: Find three fractions equivalent to $\frac{3}{5}$.

Solution:

Correct answers include $\frac{6}{10}$, $\frac{9}{15}$, and $\frac{12}{20}$.

Note:
Exercise:

Problem: Find three fractions equivalent to $\frac{4}{5}$.

Solution:

Correct answers include $\frac{8}{10}$, $\frac{12}{15}$, and $\frac{16}{20}$.

Example:
Exercise:

Problem:

Find a fraction with a denominator of 21 that is equivalent to $\frac{2}{7}$.

Solution:
Solution

To find equivalent fractions, we multiply the numerator and denominator by the same number. In this case, we need to multiply the denominator by a number that will result in 21.

Since we can multiply 7 by 3 to get 21, we can find the equivalent fraction by multiplying both the numerator and denominator by 3.

$$\frac{2}{7} = \frac{2 \cdot 3}{7 \cdot 3} = \frac{6}{21}$$

Note:
Exercise:

Problem:

Find a fraction with a denominator of 21 that is equivalent to $\frac{6}{7}$.

Solution:

$\frac{18}{21}$

Note:
Exercise:

Problem:

Find a fraction with a denominator of 100 that is equivalent to $\frac{3}{10}$.

Solution:

$\frac{30}{100}$

Simplifying Fractions

Recall the Equivalent Fractions Property:

If a, b, and c are numbers where $b \neq 0$ and $c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$

We have used this property to generate equivalent fractions where the numerator and denominator are greater than those on the original fraction. An equation always works in both directions so we can also use the property to make equivalent fractions where the numerator and denominator are less than those on the original fraction.

When we replace a fraction with an equivalent fraction that a smaller numerator and denominator than those of the original fraction, we say that we have reduced the fraction.
For example, if we replace $\frac{10}{12}$ with $\frac{5}{6}$. We can do this because $\frac{10}{12} = \frac{5 \cdot 2}{6 \cdot 2}$ and therefore the Equivalent Fractions Property applies.
Sometimes this is called reducing the fraction. Remember that while the numerator and denominator are both smaller (they have been reduced) the value of the fraction has not changed.

A fraction that has been simplified as much as possible is said to be in lowest terms.
$\frac{5}{6}$ is in lowest terms because there is no factor common to both 5 and 6. 10 and 12 had the common factor 2.

One way to find a common factor is to search for a number that will divide both the numerator and denominator without a remainder. Notice that 2 divides both 10 and 12 with no remainder.

Simplifying a fraction is related to what is commonly called canceling. For $\frac{10}{12} = \frac{5 \cdot 2}{6 \cdot 2}$ the common factor 2 is often crossed out in both the numerator and denominator. The Equivalent Fractions Property tells us that it is OK to do this.

Example:
Simplify $\frac{18}{24}$ to lowest terms.

Solution:
$\frac{18}{24} = \frac{3 \cdot 6}{4 \cdot 6}$. By the Equivalent Fractions Property that simplifies to $\frac{3}{4}$.

Exercise:

Simplify each fraction to lowest terms:

a) $\frac{15}{20}$

b) $\frac{20}{30}$

c) $\frac{7}{14}$

Problem: d) $\frac{18}{60}$

Solution:

Simplify each fraction to lowest terms:

a) $\frac{15}{20} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{3}{4}$

b) $\frac{20}{30} = \frac{2 \cdot 10}{3 \cdot 10} = \frac{2}{3}$

c) $\frac{7}{14} = \frac{1 \cdot 7}{2 \cdot 7} = \frac{1}{2}$

d) $\frac{18}{60} = \frac{3 \cdot 6}{10 \cdot 6} = \frac{1}{2}$

Model Improper Fractions and Mixed Numbers

In [link] (b), you had eight equal fifth pieces. You used five of them to make one whole, and you had three fifths left over. Let us use fraction notation to show what happened. You had eight pieces, each of them one fifth, $\frac{1}{5}$, so altogether you had eight fifths, which we can write as $\frac{8}{5}$. The fraction $\frac{8}{5}$ is one whole, 1, plus three fifths, $\frac{3}{5}$, or $1\frac{3}{5}$, which is read as *one and three-fifths*.

The number $1\frac{3}{5}$ is called a mixed number. A mixed number consists of a whole number and a fraction.

Note:
Mixed Numbers
A **mixed number** consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as follows.
Equation:

$$a\frac{b}{c} \qquad c \neq 0$$

Fractions such as $\frac{5}{4}$, $\frac{3}{2}$, $\frac{5}{5}$, and $\frac{7}{3}$ are called improper fractions. In an improper fraction, the numerator is greater than or equal to the denominator, so its value is greater than or equal to one. When a fraction has a numerator that is smaller than the denominator, it is called a proper fraction, and its value is less than one. Fractions such as $\frac{1}{2}$, $\frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

Note:
Proper and Improper Fractions
The fraction $\frac{a}{b}$ is a **proper fraction** if $a < b$ and an **improper fraction** if $a \geq b$.

Example:
Exercise:

Problem:

Name the improper fraction modeled. Then write the improper fraction as a mixed number.

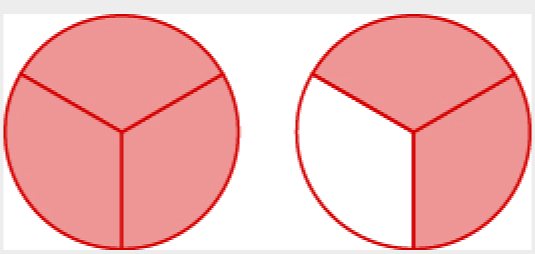
Solution:
Solution

Each circle is divided into three pieces, so each piece is $\frac{1}{3}$ of the circle. There are four pieces shaded, so there are four thirds or $\frac{4}{3}$. The figure shows that we also have one whole circle and one third, which is $1\frac{1}{3}$. So, $\frac{4}{3} = 1\frac{1}{3}$.

Note:
Exercise:

Problem:

Name the improper fraction. Then write it as a mixed number.



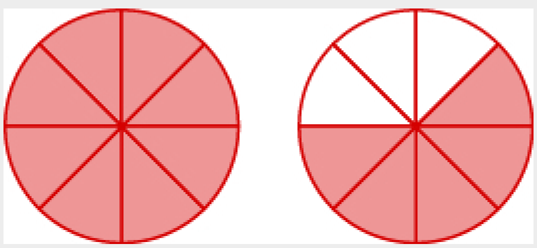
Solution:

$\frac{5}{3} = 1\frac{2}{3}$

Note:
Exercise:

Problem:

Name the improper fraction. Then write it as a mixed number.



Solution:

$\frac{13}{8} = 1\frac{5}{8}$

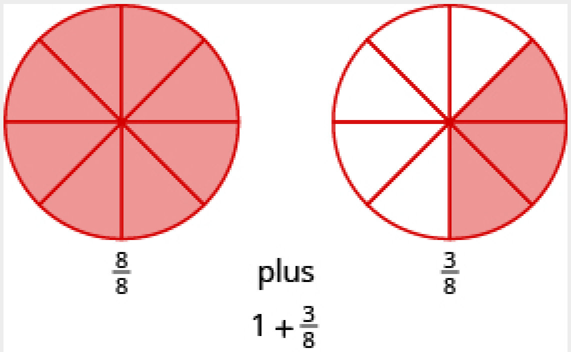
Example:
Exercise:

Problem: Draw a figure to model $\frac{11}{8}$.

Solution:
Solution

The denominator of the improper fraction is 8. Draw a circle divided into eight pieces and shade all of them. This takes care of eight

eighths, but we have 11 eighths. We must shade three of the eight parts of another circle.

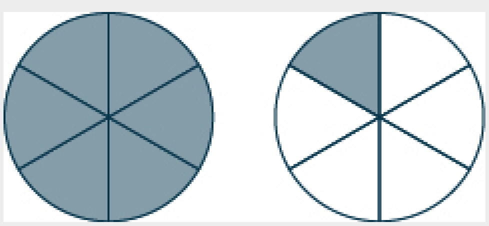


So, $\frac{11}{8} = 1\frac{3}{8}$.

Note:
Exercise:

Problem: Draw a figure to model $\frac{7}{6}$.

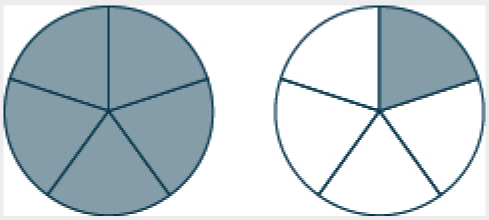
Solution:



Note:
Exercise:

Problem: Draw a figure to model $\frac{6}{5}$.

Solution:



Example:
Exercise:

Problem:

Use a model to rewrite the improper fraction $\frac{11}{6}$ as a mixed number.

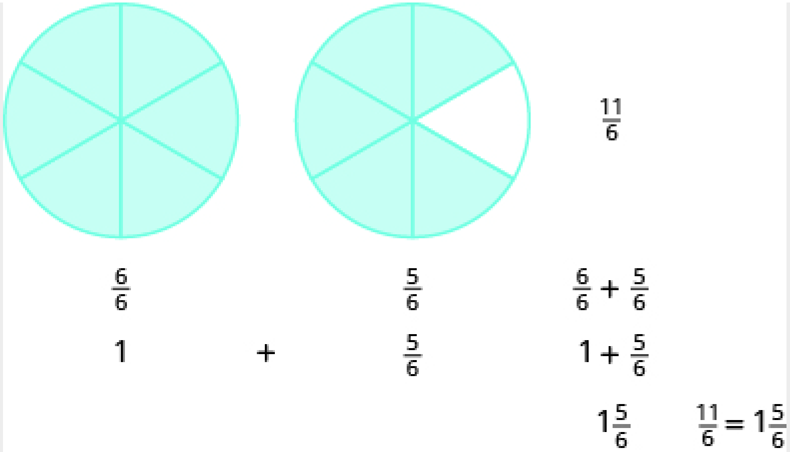
Solution:
Solution

We start with 11 sixths $\left(\frac{11}{6}\right)$. We know that six sixths makes one whole.
Equation:

$$\frac{6}{6} = 1$$

That leaves us with five more sixths, which is $\frac{5}{6}$ $\left(11 \text{ sixths minus } 6 \text{ sixths is } 5 \text{ sixths}\right)$.

So, $\frac{11}{6} = 1\frac{5}{6}$.



Note:
Exercise:

Problem:

Use a model to rewrite the improper fraction as a mixed number: $\frac{9}{7}$.

Solution:

$1\frac{2}{7}$

Note:
Exercise:

Problem:

Use a model to rewrite the improper fraction as a mixed number: $\frac{7}{4}$.

Solution:

$1\frac{3}{4}$

Example:
Exercise:

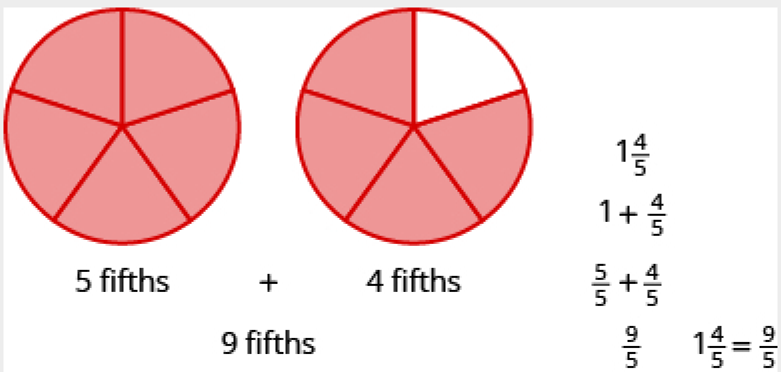
Problem:

Use a model to rewrite the mixed number $1\frac{4}{5}$ as an improper fraction.

Solution:
Solution

The mixed number $1\frac{4}{5}$ means one whole plus four fifths. The denominator is 5, so the whole is $\frac{5}{5}$. Together five fifths and four fifths equals nine fifths.

So, $1\frac{4}{5} = \frac{9}{5}$.



Note:
Exercise:

Problem:

Use a model to rewrite the mixed number as an improper fraction: $1\frac{3}{8}$.

Solution:

$\frac{11}{8}$

Note:
Exercise:

Problem:

Use a model to rewrite the mixed number as an improper fraction: $1\frac{5}{6}$.

Solution:

$\frac{11}{6}$

Convert between Improper Fractions and Mixed Numbers

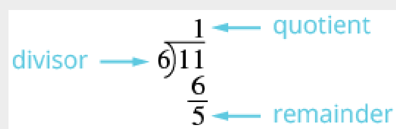
In [link], we converted the improper fraction $\frac{11}{6}$ to the mixed number $1\frac{5}{6}$ using fraction circles. We did this by grouping six sixths together to make a whole; then we looked to see how many of the 11 pieces were left. We saw that $\frac{11}{6}$ made one whole group of six sixths plus five more sixths, showing that $\frac{11}{6} = 1\frac{5}{6}$.

The division expression $\frac{11}{6}$ (which can also be written as $6\overline{)11}$) tells us to find how many groups of 6 are in 11. To convert an improper fraction to a mixed number without fraction circles, we divide.

Example:
Exercise:

Problem: Convert $\frac{11}{6}$ to a mixed number.

Solution:
Solution

	$\frac{11}{6}$
Divide the denominator into the numerator.	Remember $\frac{11}{6}$ means $11 \div 6$.
	
Identify the quotient, remainder and divisor.	
Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.	$1\frac{5}{6}$
So, $\frac{11}{6} = 1\frac{5}{6}$	

Note:
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{13}{7}$.

Solution:

$1\frac{6}{7}$.

Note:
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{14}{9}$.

Solution:

$1\frac{5}{9}$

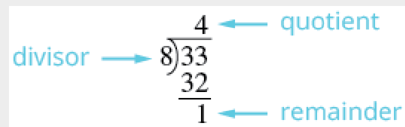
Note:
Convert an improper fraction to a mixed number.

Divide the denominator into the numerator.
Identify the quotient, remainder, and divisor.
Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.

Example:
Exercise:

Problem: Convert the improper fraction $\frac{33}{8}$ to a mixed number.

Solution:
Solution

	$\frac{33}{8}$
Divide the denominator into the numerator.	Remember, $\frac{33}{8}$ means $8\overline{)33}$.
Identify the quotient, remainder, and divisor.	
Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.	$4\frac{1}{8}$
	So, $\frac{33}{8} = 4\frac{1}{8}$

Note:
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{23}{7}$.

Solution:

$3\frac{2}{7}$

Note:
Exercise:

Problem: Convert the improper fraction to a mixed number: $\frac{48}{11}$.

Solution:

$4\frac{4}{11}$

In [link], we changed $1\frac{4}{5}$ to an improper fraction by first seeing that the whole is a set of five fifths. So we had five fifths and four more fifths.
Equation:

$$\frac{5}{5}+\frac{4}{5}=\frac{9}{5}$$

Where did the nine come from? There are nine fifths—one whole (five fifths) plus four fifths. Let us use this idea to see how to convert a mixed number to an improper fraction.

Example:
Exercise:

Problem: Convert the mixed number $4\frac{2}{3}$ to an improper fraction.

Solution:
Solution

	$4\frac{2}{3}$
Multiply the whole number by the denominator.	

The whole number is 4 and the denominator is 3.	$\frac{4 \cdot 3 + \square}{\square}$
Simplify.	$\frac{12 + \square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{12 + 2}{\square}$
Simplify.	$\frac{14}{\square}$
Write the final sum over the original denominator.	
The denominator is 3.	$\frac{14}{3}$

Note:
Exercise:

Problem: Convert the mixed number to an improper fraction: $3\,\frac{5}{7}$.

Solution:

$\frac{26}{7}$

Note:
Exercise:

Problem: Convert the mixed number to an improper fraction: $2\frac{7}{8}$.

Solution:

$\frac{23}{8}$

Note:
Convert a mixed number to an improper fraction.

Multiply the whole number by the denominator.
Add the numerator to the product found in Step 1.
Write the final sum over the original denominator.

Example:
Exercise:

Problem: Convert the mixed number $10\frac{2}{7}$ to an improper fraction.

Solution:
Solution

	$10\frac{2}{7}$
Multiply the whole number by the denominator.	
The whole number is 10 and the denominator is 7.	$\frac{10 \cdot 7 + \square}{\square}$
Simplify.	$\frac{70 + \square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{70 + 2}{\square}$
Simplify.	$\frac{72}{\square}$
Write the final sum over the original denominator.	
The denominator is 7.	$\frac{72}{7}$

Note:
Exercise:

Problem: Convert the mixed number to an improper fraction: $4\frac{6}{11}$.

Solution:

$\frac{50}{11}$

Note:
Exercise:

Problem: Convert the mixed number to an improper fraction: $11\frac{1}{3}$.

Solution:

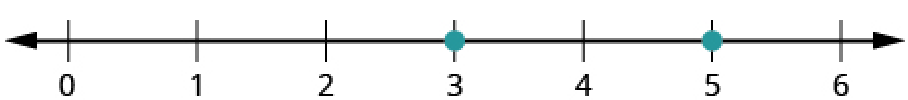
$\frac{34}{3}$

Locate Fractions and Mixed Numbers on the Number Line

Now we are ready to plot fractions on a number line. This will help us visualize fractions and understand their values.

Let us locate $\frac{1}{5}$, $\frac{4}{5}$, 3, $3\frac{1}{3}$, $\frac{7}{4}$, $\frac{9}{2}$, 5, and $\frac{8}{3}$ on the number line.

We will start with the whole numbers 3 and 5 because they are the easiest to plot.



The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$. We know proper fractions have values less than one, so $\frac{1}{5}$ and $\frac{4}{5}$ are located between the whole numbers 0

and 1. The denominators are both 5, so we need to divide the segment of the number line between 0 and 1 into five equal parts. We can do this by drawing four equally spaced marks on the number line, which we can then label as $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$.

Now plot points at $\frac{1}{5}$ and $\frac{4}{5}$.

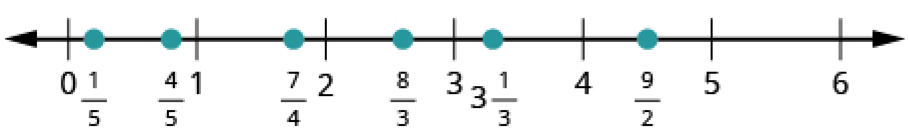
The only mixed number to plot is $3\frac{1}{3}$. Between what two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than 3, but not a whole unit greater, $3\frac{1}{3}$ is between 3 and 4. We need to divide the portion of the number line between 3 and 4 into three equal pieces (thirds) and plot $3\frac{1}{3}$ at the first mark.

Finally, look at the improper fractions $\frac{7}{4}$, $\frac{9}{2}$, and $\frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

Equation:

$$\frac{7}{4} = 1\frac{3}{4}, \quad \frac{9}{2} = 4\frac{1}{2}, \quad \frac{8}{3} = 2\frac{2}{3}$$

Here is the number line with all the points plotted.



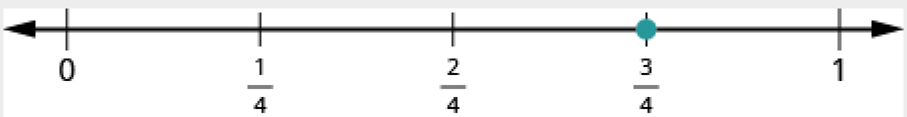
Example:
Exercise:

Problem:

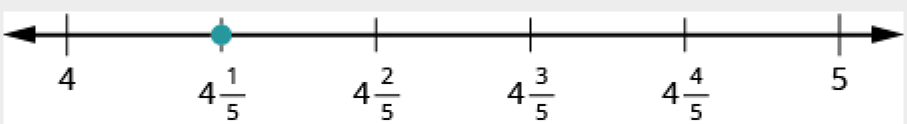
Locate and label the following on a number line: $\frac{3}{4}$, $\frac{4}{3}$, $\frac{5}{3}$, $4\frac{1}{5}$, and $\frac{7}{2}$.

Solution:
Solution

Start by locating the proper fraction $\frac{3}{4}$. It is between 0 and 1. To do this, divide the distance between 0 and 1 into four equal parts. Then plot $\frac{3}{4}$.



Next, locate the mixed number $4\frac{1}{5}$. It is between 4 and 5 on the number line. Divide the number line between 4 and 5 into five equal parts, and then plot $4\frac{1}{5}$ one-fifth of the way between 4 and 5.



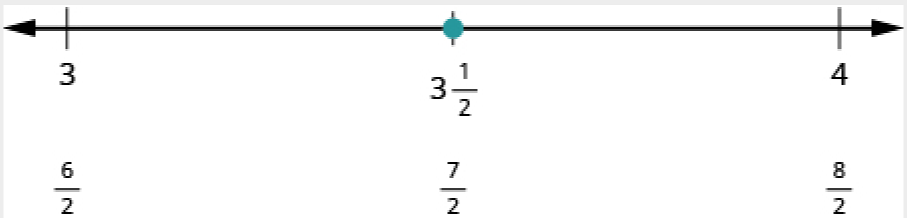
Now locate the improper fractions $\frac{4}{3}$ and $\frac{5}{3}$.

It is easier to plot them if we convert them to mixed numbers first.
Equation:

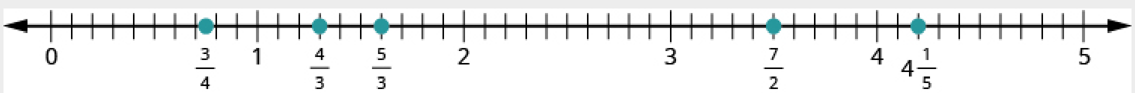
$$\frac{4}{3} = 1\frac{1}{3}, \quad \frac{5}{3} = 1\frac{2}{3}$$

Divide the distance between 1 and 2 into thirds.

Next let us plot $\frac{7}{2}$. We write it as a mixed number, $\frac{7}{2} = 3\frac{1}{2}$. Plot it between 3 and 4.



The number line shows all the numbers located on the number line.

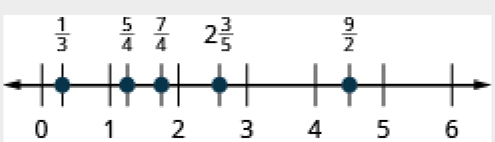


Note:
Exercise:

Problem:

Locate and label the following on a number line: $\frac{1}{3}, \frac{5}{4}, \frac{7}{4}, 2\frac{3}{5}, \frac{9}{2}$.

Solution:

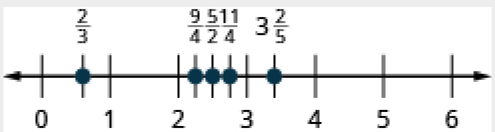


Note:
Exercise:

Problem:

Locate and label the following on a number line: $\frac{2}{3}$, $\frac{5}{2}$, $\frac{9}{4}$, $\frac{11}{4}$, $3\frac{2}{5}$.

Solution:



Key Concepts

- **Property of One**
    - Any number, except zero, divided by itself is one.
      $\frac{a}{a} = 1$, where $a \neq 0$.
- **Equivalent Fractions Property**
    - If a, b, and c are numbers where $b \neq 0$, $c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$.
- **Mixed Numbers**
    - A **mixed number** consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$.
    - It is written as follows: $a\frac{b}{c} \qquad c \neq 0$
- **Proper and Improper Fractions**

- The fraction ab is a proper fraction if $a < b$ and an improper fraction if $a \geq b$.

- **Convert an improper fraction to a mixed number.**

  Divide the denominator into the numerator.
  Identify the quotient, remainder, and divisor.
  Write the mixed number as quotient $\frac{\text{remainder}}{\text{divisor}}$.

- **Convert a mixed number to an improper fraction.**

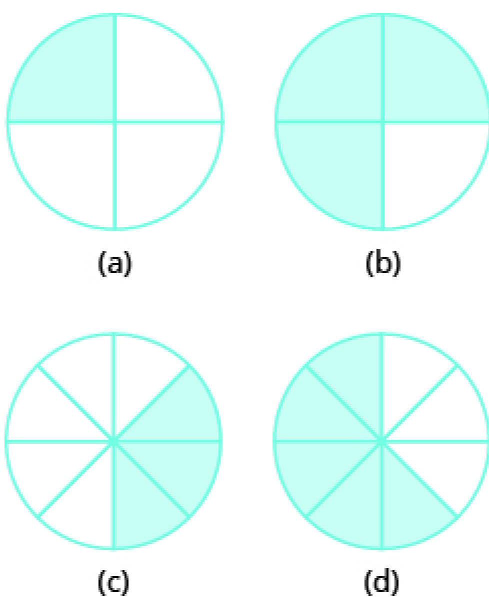
  Multiply the whole number by the denominator.
  Add the numerator to the product found in Step 1.
  Write the final sum over the original denominator.

Exercises

Practice Makes Perfect

In the following exercises, name the fraction of each figure that is shaded.
Exercise:

Problem:

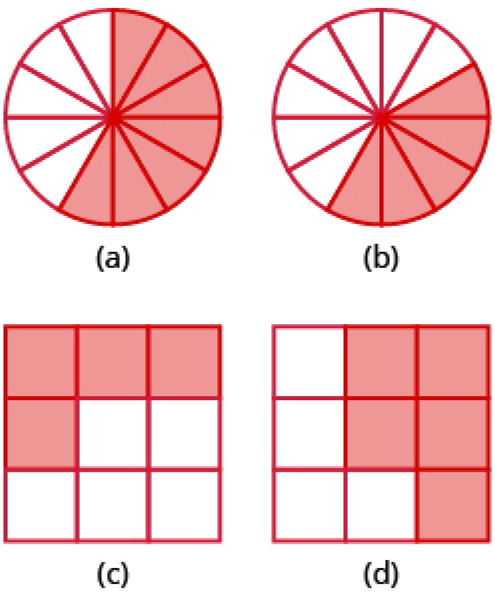


Solution:

ⓐ $\frac{1}{4}$
ⓑ $\frac{3}{4}$
ⓒ $\frac{3}{8}$
ⓓ $\frac{5}{9}$

Exercise:

Problem:

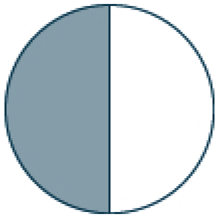


In the following exercises, shade parts of circles or squares to model the following fractions.

Exercise:

Problem: $\frac{1}{2}$

Solution:



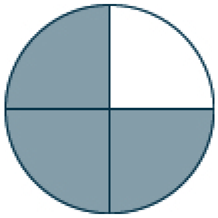
Exercise:

Problem: $\frac{1}{3}$

Exercise:

Problem: $\frac{3}{4}$

Solution:



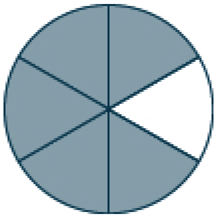
Exercise:

Problem: $\frac{2}{5}$

Exercise:

Problem: $\frac{5}{6}$

Solution:



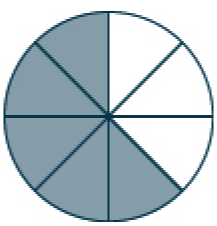
Exercise:

Problem: $\frac{7}{8}$

Exercise:

Problem: $\frac{5}{8}$

Solution:



Exercise:

Problem: $\frac{7}{10}$

In the following exercises, use fraction towers or draw a figure to find equivalent fractions.

Exercise:

Problem: How many sixths equal one-third?

Exercise:

Problem: How many twelfths equal one-third?

Solution:

4

Exercise:

Problem: How many eighths equal three-fourths?

Exercise:

Problem: How many twelfths equal three-fourths?

Solution:

9

Exercise:

Problem: How many fourths equal three-halves?

Exercise:

Problem: How many sixths equal three-halves?

Solution:

9

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

Exercise:

Problem: $\frac{1}{4}$

Exercise:

Problem: $\frac{1}{3}$

Solution:

Answers may vary. Correct answers include $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}$.

Exercise:

Problem: $\frac{3}{8}$

Exercise:

Problem: $\frac{5}{6}$

Solution:

Answers may vary. Correct answers include $\frac{10}{12}, \frac{15}{18}, \frac{20}{24}$.

Exercise:

Problem: $\frac{2}{7}$

Exercise:

Problem: $\frac{5}{9}$

Solution:

Answers may vary. Correct answers include $\frac{10}{18}, \frac{15}{27}, \frac{20}{36}$.

In the following exercises, simplify as much as possible.

Exercise:

Problem: $\frac{9}{12}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{8}{20}$

Solution:

$\frac{2}{5}$

Exercise:

Problem: $\frac{11}{22}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{28}{63}$

Solution:

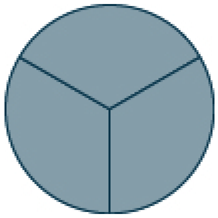
$\frac{4}{9}$

In the following exercises, use fraction circles to make wholes, if possible, with the following pieces.

Exercise:

Problem: 3 thirds

Solution:



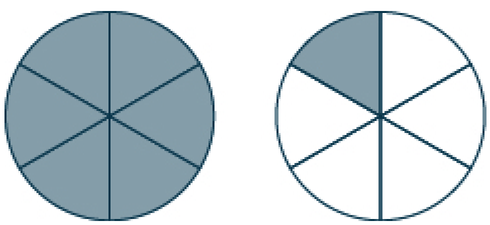
Exercise:

Problem: 8 eighths

Exercise:

Problem: 7 sixths

Solution:



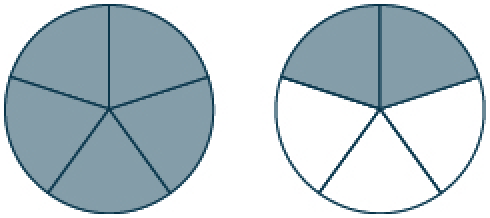
Exercise:

Problem: 4 thirds

Exercise:

Problem: 7 fifths

Solution:



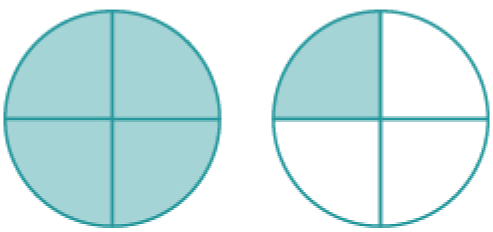
Exercise:

Problem: 7 fourths

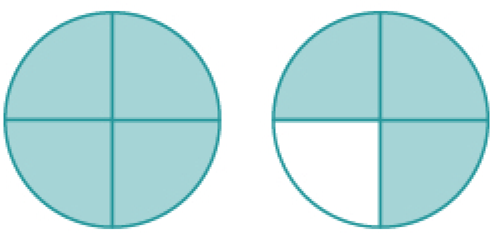
In the following exercises, name the improper fractions. Then write each improper fraction as a mixed number.

Exercise:

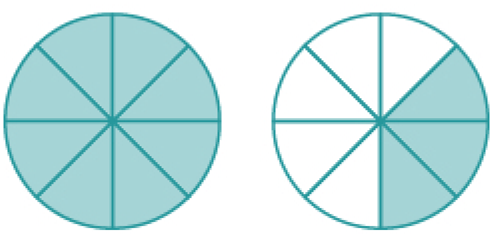
Problem:



(a)



(b)



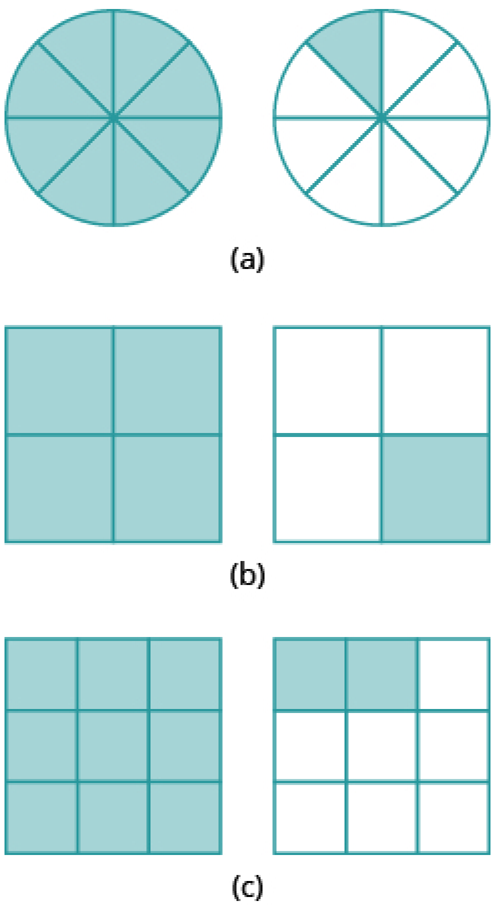
(c)

Solution:

ⓐ $\frac{5}{4} = 1\frac{1}{4}$
ⓑ $\frac{7}{4} = 1\frac{3}{4}$
ⓒ $\frac{11}{8} = 1\frac{3}{8}$

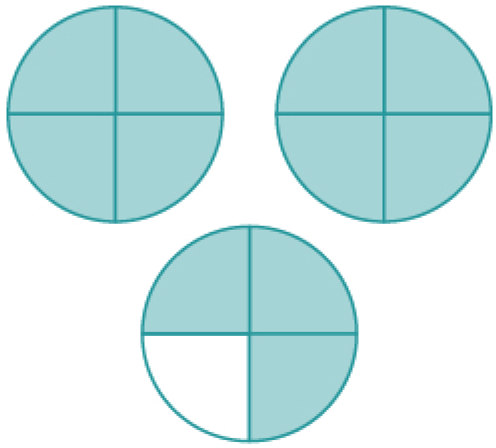
Exercise:

Problem:

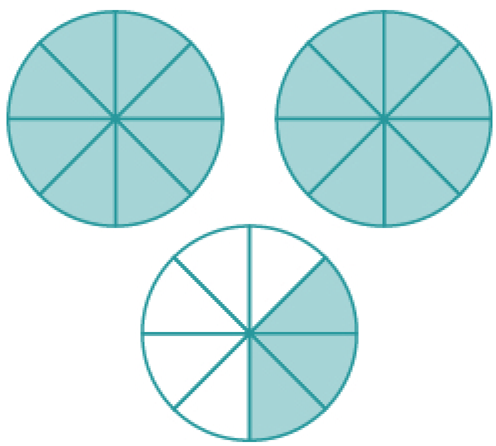


Exercise:

Problem:



(a)



(b)

Solution:

ⓐ $\frac{11}{4} = 2\frac{3}{4}$
ⓑ $\frac{19}{8} = 2\frac{3}{8}$

In the following exercises, draw fraction circles to model the given fraction.

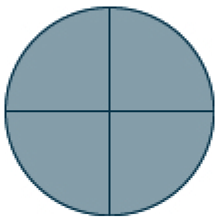
Exercise:

Problem: $\frac{3}{3}$

Exercise:

Problem: $\frac{4}{4}$

Solution:



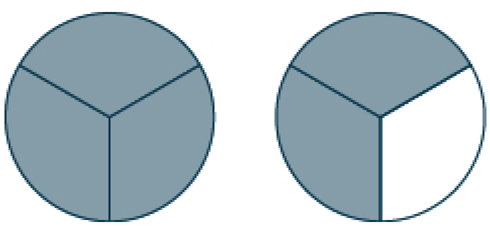
Exercise:

Problem: $\frac{7}{4}$

Exercise:

Problem: $\frac{5}{3}$

Solution:



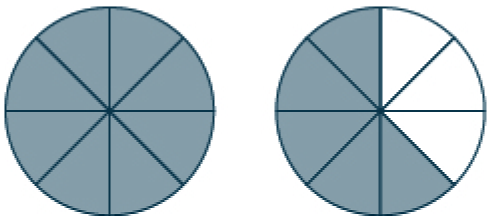
Exercise:

Problem: $\frac{11}{6}$

Exercise:

Problem: $\frac{13}{8}$

Solution:



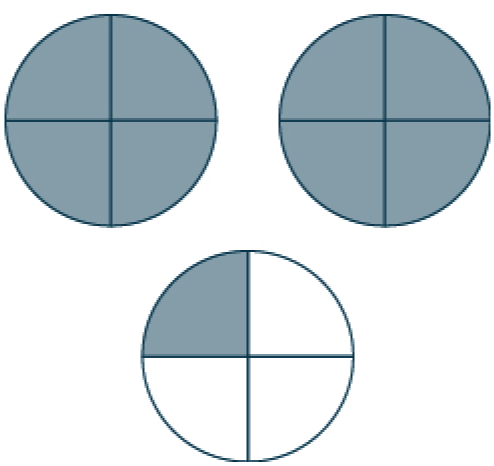
Exercise:

Problem: $\frac{10}{3}$

Exercise:

Problem: $\frac{9}{4}$

Solution:



In the following exercises, rewrite the improper fraction as a mixed number.

Exercise:

Problem: $\frac{3}{2}$

Exercise:

Problem: $\frac{5}{3}$

Solution:

$1\frac{2}{3}$

Exercise:

Problem: $\frac{11}{4}$

Exercise:

Problem: $\frac{13}{5}$

Solution:

$2\frac{3}{5}$

Exercise:

Problem: $\frac{25}{6}$

Exercise:

Problem: $\frac{28}{9}$

Solution:

$3\frac{1}{9}$

Exercise:

Problem: $\frac{42}{13}$

Exercise:

Problem: $\frac{47}{15}$

Solution:

$3\,\frac{2}{15}$

In the following exercises, rewrite the mixed number as an improper fraction.

Exercise:

Problem: $1\,\frac{2}{3}$

Exercise:

Problem: $1\,\frac{2}{5}$

Solution:

$\frac{7}{5}$

Exercise:

Problem: $2\,\frac{1}{4}$

Exercise:

Problem: $2\,\frac{5}{6}$

Solution:

$\frac{17}{6}$

Exercise:

Problem: $2\frac{7}{9}$

Exercise:

Problem: $2\frac{5}{7}$

Solution:

$\frac{19}{7}$

Exercise:

Problem: $3\frac{4}{7}$

Exercise:

Problem: $3\frac{5}{9}$

Solution:

$\frac{32}{9}$

In the following exercises, plot the numbers on a number line.

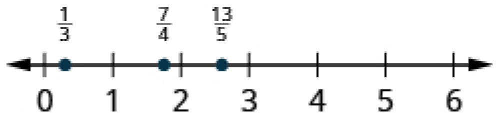
Exercise:

Problem: $\frac{2}{3}, \frac{5}{4}, \frac{12}{5}$

Exercise:

Problem: $\frac{1}{3}, \frac{7}{4}, \frac{13}{5}$

Solution:



Exercise:

Problem: $\frac{1}{4}$, $\frac{9}{5}$, $\frac{11}{3}$

Exercise:

Problem: $\frac{7}{10}$, $\frac{5}{2}$, $\frac{13}{8}$, 3

Solution:

Everyday Math

Exercise:

Problem:

Music Measures A choreographed dance is broken into counts. A $\frac{1}{1}$ count has one step in a count, a $\frac{1}{2}$ count has two steps in a count and a $\frac{1}{3}$ count has three steps in a count. How many steps would be in a $\frac{1}{5}$ count? What type of count has four steps in it?

Writing Exercises

Exercise:

Problem:

Give an example from your life experience (outside of school) where it was important to understand fractions.

Solution:

Answers will vary.

Exercise:

Problem:

Explain how you locate the improper fraction $\frac{21}{4}$ on a number line on which only the whole numbers from 0 through 10 are marked.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
understand the meaning of fractions.			
model improper fractions and mixed numbers.			
convert between improper fractions and mixed numbers.			
model equivalent fractions.			
find equivalent fractions.			
locate fractions and mixed numbers on the number line.			
order fractions and mixed numbers.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

equivalent fractions
: Equivalent fractions are two or more fractions that have the same value.

fraction

A fraction is written $\frac{a}{b}$. in a fraction, a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

mixed number

A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as $a\frac{b}{c}$, where $c \neq 0$.

proper and improper fractions

The fraction $\frac{a}{b}$ is *proper* if $a < b$and *improper* if $a > b$.

Add and Subtract Fractions with Common Denominators Beginning Level
By the end of this section, you will be able to:

- Model fraction addition
- Add fractions with a common denominator
- Model fraction subtraction
- Subtract fractions with a common denominator

Note:
Before you get started, take this readiness quiz.

1. Draw a model of the fraction $\frac{3}{4}$.
   If you missed this problem, review [link].

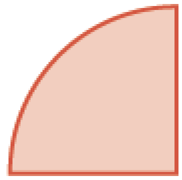
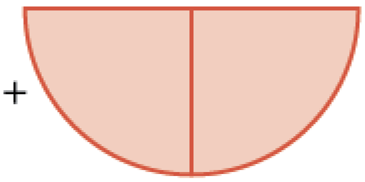
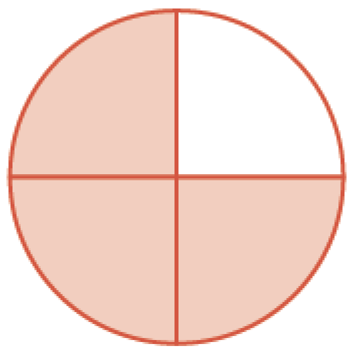
Model Fraction Addition

How many quarters are pictured? One quarter plus 2 quarters equals 3 quarters.

Remember, quarters are really fractions of a dollar. Quarters are another way to say fourths. So the picture of the coins shows that
Equation:

$$\begin{array}{ccccc} \frac{1}{4} & & \frac{2}{4} & & \frac{3}{4} \\ \text{one quarter} & + & \text{two quarters} & = & \text{three quarters} \end{array}$$

Let’s use fraction circles to model the same example, $\frac{1}{4} + \frac{2}{4}$.

Start with one $\frac{1}{4}$ piece.		$\frac{1}{4}$
Add two more $\frac{1}{4}$ pieces.	+ 	$+\ \frac{2}{4}$
The result is $\frac{3}{4}$.		$\frac{3}{4}$

So again, we see that

Equation:

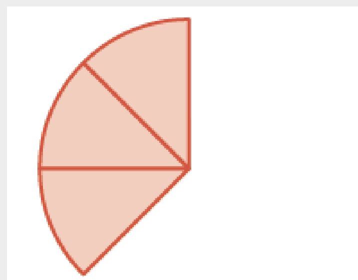
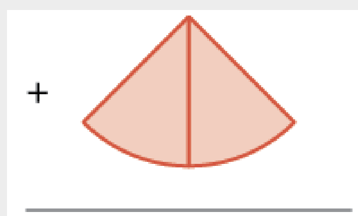
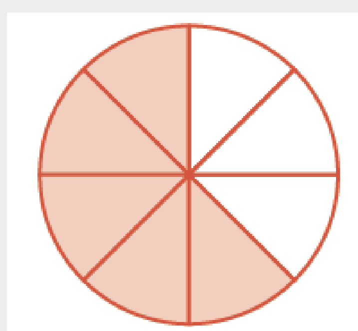
$$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$$

Example:

Exercise:

Problem: Use a model to find the sum $\frac{3}{8}+\frac{2}{8}$.

Solution:
Solution

Start with three $\frac{1}{8}$ pieces.		$\frac{3}{8}$
Add two $\frac{1}{8}$ pieces.		$+\frac{2}{8}$
How many $\frac{1}{8}$ pieces are there?		$\frac{5}{8}$

There are five $\frac{1}{8}$ pieces, or five-eighths. The model shows that $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$.

Note:
Exercise:

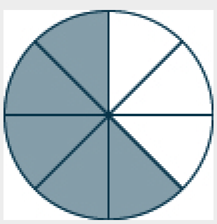
Problem:

Use a model to find each sum. Show a diagram to illustrate your model.

$\frac{1}{8} + \frac{4}{8}$

Solution:

$\frac{5}{8}$



Note:
Exercise:

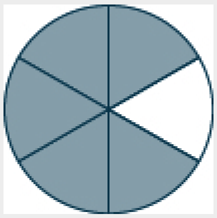
Problem:

Use a model to find each sum. Show a diagram to illustrate your model.

$\frac{1}{6} + \frac{4}{6}$

Solution:

$\frac{5}{6}$



Add Fractions with a Common Denominator

[link] shows that to add the same-size pieces—meaning that the fractions have the same denominator—we just add the number of pieces.

Note:
Fraction Addition
If $a, b,$ and c are numbers where $c \neq 0$, then
Equation:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

To add fractions with a common denominators, add the numerators and place the sum over the common denominator.

Example:
Exercise:

Problem: Find the sum: $\frac{3}{5} + \frac{1}{5}$.

Solution:
Solution

	$\frac{3}{5}+\frac{1}{5}$
Add the numerators and place the sum over the common denominator.	$\frac{3+1}{5}$
Simplify.	$\frac{4}{5}$

Note:
Exercise:

Problem: Find each sum: $\frac{3}{6}+\frac{2}{6}$.

Solution:

$\frac{5}{6}$

Note:
Exercise:

Problem: Find each sum: $\frac{3}{10}+\frac{7}{10}$.

Solution:

1

Example:
Exercise:

Problem: Find the sum: $\frac{3}{12} + \frac{5}{12}$.

Solution:
Solution

	$\frac{3}{12} + \frac{5}{12}$
Add the numerators and place the sum over the common denominator.	$\frac{3+5}{12}$
Add.	$\frac{8}{12}$
Simplify the fraction.	$\frac{2}{3}$

Note:
Exercise:

Problem: Find each sum: $\frac{2}{15}+\frac{7}{15}$.

Solution:

$\frac{3}{5}$

Model Fraction Subtraction

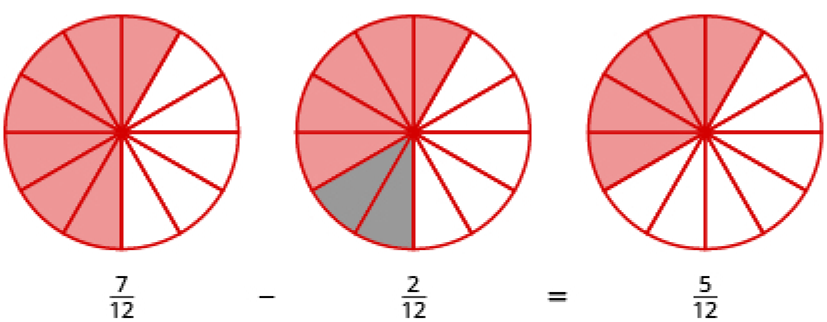
Subtracting two fractions with common denominators is much like adding fractions. Think of a pizza that was cut into 12 slices. Suppose five pieces are eaten for dinner. This means that, after dinner, there are seven pieces (or $\frac{7}{12}$ of the pizza) left in the box. If Leonardo eats 2 of these remaining pieces (or $\frac{2}{12}$ of the pizza), how much is left? There would be 5 pieces left (or $\frac{5}{12}$ of the pizza).

Equation:

$$\frac{7}{12}-\frac{2}{12}=\frac{5}{12}$$

Let's use fraction circles to model the same example, $\frac{7}{12}-\frac{2}{12}$.

Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces. How many twelfths are left?



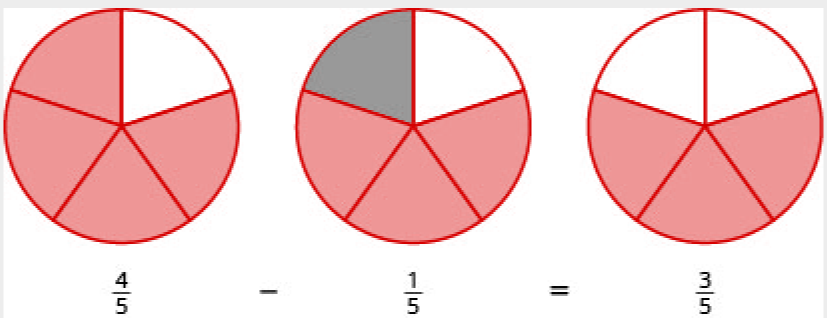
Again, we have five twelfths, $\frac{5}{12}$.

Example:
Exercise:

Problem: Use fraction circles to find the difference: $\frac{4}{5} - \frac{1}{5}$.

Solution:
Solution

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece. Count how many fifths are left. There are three $\frac{1}{5}$ pieces left.



Note:
Exercise:

Problem:

Use a model to find each difference. Show a diagram to illustrate your model.

$\frac{7}{8} - \frac{4}{8}$

Solution:

$\frac{3}{8}$, models may differ.

Note:
Exercise:

Problem:

Use a model to find each difference. Show a diagram to illustrate your model.

$\frac{5}{6} - \frac{4}{6}$

Solution:

$\frac{1}{6}$, models may differ

Subtract Fractions with a Common Denominator

We subtract fractions with a common denominator in much the same way as we add fractions with a common denominator.

Note:
Fraction Subtraction
If a, b, and c are numbers where $c \neq 0$, then
Equation:

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

To subtract fractions with a common denominators, we subtract the numerators and place the difference over the common denominator.

Example:
Exercise:

Problem: Find the difference: $\frac{23}{24} - \frac{14}{24}$.

Solution:
Solution

	$\frac{23}{24} - \frac{14}{24}$
Subtract the numerators and place the difference over the common denominator.	$\frac{23-14}{24}$
Simplify the numerator.	$\frac{9}{24}$
Simplify the fraction by removing common factors.	$\frac{3}{8}$

Note:
Exercise:

Problem: Find the difference: $\frac{19}{28} - \frac{7}{28}$.

Solution:

$\frac{3}{7}$

Note:
Exercise:

Problem: Find the difference: $\frac{27}{32} - \frac{11}{32}$.

Solution:

$\frac{1}{2}$

Now lets do an example that involves both addition and subtraction.

Example:
Exercise:

Problem: Simplify: $\frac{5}{8} - \frac{3}{8} - \frac{1}{8}$.

Solution:
Solution

	$\frac{5}{8} - \frac{3}{8} - \frac{1}{8}$
Combine the numerators over the common denominator.	$\frac{5-3-1}{8}$
Simplify the numerator, working left to right.	$\frac{2-1}{8}$
Subtract the terms in the numerator.	$\frac{1}{8}$

Note:
Exercise:

Problem: Simplify: $\frac{4}{5} - \frac{2}{5} + \frac{3}{5}$.

Solution:

1

Note:
Exercise:

Problem: Simplify: $\frac{5}{9} - \frac{4}{9} + \frac{5}{9}$.

Solution:

$\frac{2}{3}$

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Adding Fractions With Pattern Blocks
- Adding Fractions With Like Denominators
- Subtracting Fractions With Like Denominators

Key Concepts

- **Fraction Addition**
  - If a, b, and c are numbers where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+c}{c}$.
  - To add fractions, add the numerators and place the sum over the common denominator.
- **Fraction Subtraction**
  - If a, b, and c are numbers where $c \neq 0$, then $\frac{a}{c} - \frac{b}{c} = \frac{a-c}{c}$.
  - To subtract fractions, subtract the numerators and place the difference over the common denominator.

Exercises

Practice Makes Perfect

Model Fraction Addition

In the following exercises, use a model to add the fractions. Show a diagram to illustrate your model.

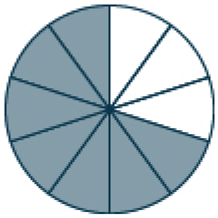
Exercise:

Problem: $\frac{2}{5} + \frac{1}{5}$

Exercise:

Problem: $\frac{3}{10} + \frac{4}{10}$

Solution:



$\frac{7}{10}$

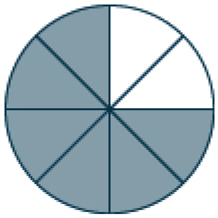
Exercise:

Problem: $\frac{1}{6} + \frac{3}{6}$

Exercise:

Problem: $\frac{3}{8} + \frac{3}{8}$

Solution:



$\frac{3}{4}$

Add Fractions with a Common Denominator

In the following exercises, find each sum.

Exercise:

Problem: $\frac{4}{9} + \frac{1}{9}$

Exercise:

Problem: $\frac{2}{9} + \frac{5}{9}$

Solution:

$\frac{7}{9}$

Exercise:

Problem: $\frac{6}{13} + \frac{7}{13}$

Exercise:

Problem: $\frac{9}{15} + \frac{7}{15}$

Solution:

$\frac{16}{15}$

Exercise:

Problem: $\frac{1}{8} + \frac{5}{8}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{5}{16} + \frac{9}{16}$

Solution:

$\frac{7}{8}$

Exercise:

Problem: $\frac{17}{19} - \frac{9}{19}$

Solution:

$\frac{8}{19}$

Exercise:

Problem: $\frac{5}{12} + \frac{2}{12} + \frac{3}{12}$

Solution:

$\frac{5}{6}$

Model Fraction Subtraction

In the following exercises, use a model to subtract the fractions. Show a diagram to illustrate your model.

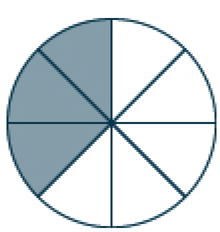
Exercise:

Problem: $\frac{5}{8} - \frac{2}{8}$

Exercise:

Problem: $\frac{5}{6} - \frac{2}{6}$

Solution:



$\frac{1}{2}$

Subtract Fractions with a Common Denominator

In the following exercises, find the difference.

Exercise:

Problem: $\frac{4}{5} - \frac{1}{5}$

Exercise:

Problem: $\frac{4}{5} - \frac{3}{5}$

Solution:

$\frac{1}{5}$

Exercise:

Problem: $\frac{11}{15} - \frac{7}{15}$

Exercise:

Problem: $\frac{9}{13} - \frac{4}{13}$

Solution:

$\frac{5}{13}$

Exercise:

Problem: $\frac{11}{12} - \frac{5}{12}$

Exercise:

Problem: $\frac{7}{12} - \frac{5}{12}$

Solution:

$\frac{1}{6}$

Exercise:

Problem: $\frac{13}{7} - \frac{11}{7}$

Solution:

$\frac{2}{7}$

Exercise:

Problem: $\frac{8}{11} - \frac{5}{11}$

Solution:

$\frac{3}{11}$

Everyday Math

Exercise:

Problem:

Trail Mix Jacob is mixing together nuts and raisins to make trail mix. He has $\frac{6}{10}$ of a pound of nuts and $\frac{3}{10}$ of a pound of raisins. How much trail mix will that make?

Exercise:

Problem:

Baking Janet needs $\frac{5}{8}$ of a cup of flour for a recipe she is making. She only has $\frac{3}{8}$ of a cup of flour and will ask to borrow the rest from her next-door neighbor. How much flour does she have to borrow?

Solution:

$\frac{1}{4}$ cup

Writing Exercises

Exercise:

Problem:

Greg dropped his case of drill bits and three of the bits fell out. The case has slots for the drill bits, and the slots are arranged in order from smallest to largest. Greg needs to put the bits that fell out back in the case in the empty slots. Where do the three bits go? Explain how you know.
Bits in case: $\frac{1}{16}, \frac{1}{8}, \_\_\_, \_\_\_, \frac{5}{16}, \frac{3}{8}, \_\_\_, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}.$
Bits that fell out: $\frac{7}{16}, \frac{3}{16}, \frac{1}{4}.$

Exercise:

Problem:

After a party, Lupe has $\frac{5}{12}$ of a cheese pizza, $\frac{4}{12}$ of a pepperoni pizza, and $\frac{4}{12}$ of a veggie pizza left. Will all the slices fit into 1 pizza box? Explain your reasoning.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
model fraction addition.			
add fractions with a common denominator.			
model fraction subtraction.			
subtract fractions with a common denominator.			
find the least common denominator (LCD).			
convert fractions to equivalent fractions with the LCD.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Add and Subtract Fractions with Different Denominators Beginning Level
By the end of this section, you will be able to:

- Find the least common denominator (LCD)
- Convert fractions to equivalent fractions with the LCD
- Add and subtract fractions with different denominators

Note:
Before you get started, take this readiness quiz.

1. Find two fractions equivalent to $\frac{5}{6}$.
   If you missed this problem, review [link].
2. Simplify: $\frac{5}{8}-\frac{3}{8}-\frac{1}{8}$.
   If you missed this problem, review [link].

Find the Least Common Denominator

In the previous section, we explained how to add and subtract fractions with a common denominator. But how can we add and subtract fractions with unlike denominators?

Let’s think about coins again. Can you add one quarter and one dime? You could say there are two coins, but that’s not very useful. To find the total value of one quarter plus one dime, you change them to the same kind of unit—cents. One quarter equals 25 cents and one dime equals 10 cents, so the sum is 35 cents. See [link].

Together, a quarter and a dime are worth 35 cents, or $\frac{35}{100}$ of a dollar.

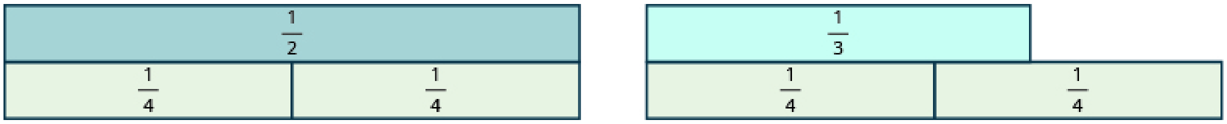
Similarly, when we add fractions with different denominators we have to convert them to equivalent fractions with a common denominator. With the coins, when we convert to cents, the denominator is 100. Since there are 100 cents in one dollar, 25 cents is $\frac{25}{100}$ and 10 cents is $\frac{10}{100}$. So we add $\frac{25}{100} + \frac{10}{100}$ to get $\frac{35}{100}$, which is 35 cents.

You have practiced adding and subtracting fractions with common denominators. Now let’s see what you need to do with fractions that have different denominators.

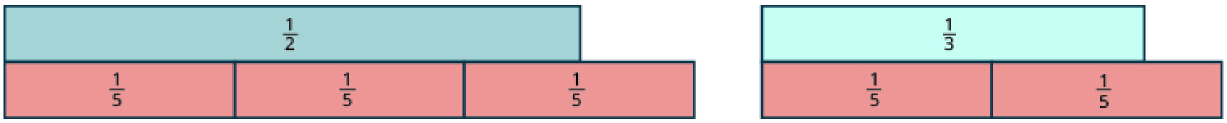
First, we will use fraction towers to model finding the common denominator of $\frac{1}{2}$ and $\frac{1}{3}$.

We’ll start with one $\frac{1}{2}$ tower and $\frac{1}{3}$ tower. We want to find a common fraction tower that we can use to match *both* $\frac{1}{2}$ and $\frac{1}{3}$ exactly.

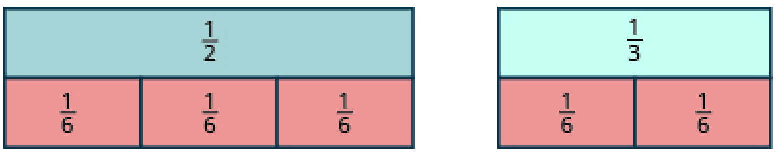
If we try the $\frac{1}{4}$ pieces, 2 of them exactly match the $\frac{1}{2}$ piece, but they do not exactly match the $\frac{1}{3}$ piece.



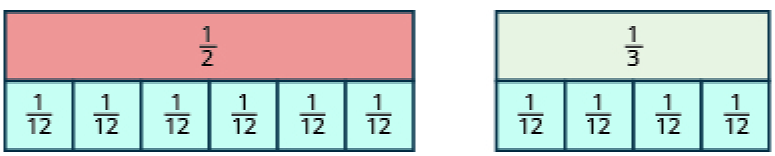
If we try the $\frac{1}{5}$ pieces, they do not exactly match the $\frac{1}{2}$ piece or the $\frac{1}{3}$ piece.



If we try the $\frac{1}{6}$ pieces, we see that exactly 3 of them match the $\frac{1}{2}$ piece, and exactly 2 of them match the $\frac{1}{3}$ piece.



If we were to try the $\frac{1}{12}$ pieces, they would also work.



Even smaller pieces, such as $\frac{1}{24}$ and $\frac{1}{48}$, would also exactly match the $\frac{1}{2}$ piece and the $\frac{1}{3}$ piece.

The denominator of the largest piece that matches both fractions is the **least common denominator (LCD)** of the two fractions. Recall that for the same numerator, the smaller the denominator then the larger the piece will be.

The least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

Notice that all of the pieces that exactly match $\frac{1}{2}$ and $\frac{1}{3}$ have something in common: Their denominators are common multiples of 2 and 3, the denominators of $\frac{1}{2}$ and $\frac{1}{3}$.

The least common multiple (LCM) of the denominators is 6, and so we say that 6 is the least common denominator (LCD) of the fractions $\frac{1}{2}$ and $\frac{1}{3}$.

Note:
Least Common Denominator
The **least common denominator (LCD)** of two fractions is the least common multiple (LCM) of their denominators.

To find the LCD of two fractions, we will find the LCM of their denominators. One procedure to find the LCM is shown below. We only use the denominators of the fractions, not the numerators, when finding the LCD.

Example:
Exercise:

Problem: Find the LCD for the fractions $\frac{5}{6}$ and $\frac{3}{10}$.

Solution:
Solution

Start a list of multiples for each denominator.	6, 12, 18, 24, 30, 36 10, 20, 30, 40

Scan the lists to see the smallest value they have in common.	30
The LCM of 6 and 10 is 30, so the LCD of $\frac{5}{6}$ and $\frac{3}{10}$ is 30.	LCD of $\frac{5}{6}$ and $\frac{3}{10}$ is 30.

Note:
Exercise:

Problem:

Find the least common denominator for the fractions: $\frac{7}{12}$ and $\frac{11}{15}$.

Solution:

60

Note:
Exercise:

Problem:

Find the least common denominator for the fractions: $\frac{13}{15}$ and $\frac{17}{5}$.

Solution:

15

To find the LCD of two fractions, find the LCM of their denominators.

Note:
Find the least common denominator (LCD) of two fractions.

Start a list of multiples for each denominator.
Scan the list starting at the low end to see if they have a common multiple.
The first common multiple found is the LCM.
If no common multiple is found increase the list with the smallest last value.
Check to see if the new value is a common multiple.
If not repeat adding a value to the list with the smallest value and check again.

Example:
Exercise:

Problem:

Find the least common denominator for the fractions $\frac{8}{15}$ and $\frac{11}{12}$.

Solution:
Solution

To find the LCD, we find the LCM of the denominators.

Find the LCM of 15 and 12.

15, 30, 45
12, 24, 36
No common multiple so far

15, 30, 45
12, 24, 36, 48
No common multiple so far

15, 30, 45, 60

12, 24, 36, 48
No common multiple so far

15, 30, 45, 60
12, 24, 36, 48, 60

LCM = 60.

The LCM of 15 and 12 is 60. So, the LCD of $\frac{8}{15}$ and $\frac{11}{12}$ is 60.

Note:
Exercise:

Problem:

Find the least common denominator for the fractions: $\frac{3}{12}$ and $\frac{5}{8}$.

Solution:

24

Note:
Exercise:

Problem:

Find the least common denominator for the fractions: $\frac{9}{14}$ and $\frac{21}{10}$.

Solution:

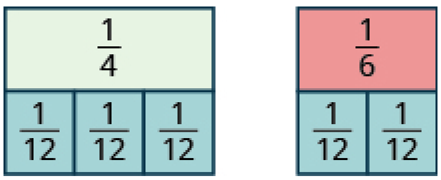
70

Convert Fractions to Equivalent Fractions with the LCD

Earlier, we used fraction tiles to see that the LCD of $\frac{1}{4}$ and $\frac{1}{6}$ is 12. We saw that three $\frac{1}{12}$ pieces exactly matched $\frac{1}{4}$ and two $\frac{1}{12}$ pieces exactly matched $\frac{1}{6}$, so

Equation:

$$\frac{1}{4}=\frac{3}{12}\text{and}\frac{1}{6}=\frac{2}{12}.$$



We say that $\frac{1}{4}$ and $\frac{3}{12}$ are equivalent fractions and also that $\frac{1}{6}$ and $\frac{2}{12}$ are equivalent fractions.

We can use the Equivalent Fractions Property to algebraically change a fraction to an equivalent one. Remember, two fractions are equivalent if they have the same value. The Equivalent Fractions Property is repeated below for reference.

Note:
Equivalent Fractions Property
If a,b,c are whole numbers where $b\ne 0,c\ne 0,$ then
Equation:

$$\frac{a}{b}=\frac{a\cdot c}{b\cdot c}\quad\text{and}\quad\frac{a\cdot c}{b\cdot c}=\frac{a}{b}$$

To add or subtract fractions with different denominators, we will first have to convert each fraction to an equivalent fraction with the LCD. Let's see how to change $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12 without using models.

Example:
Exercise:

Problem:

Convert $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12, their LCD.

Solution:
Solution

Find the LCD.	The LCD of $\frac{1}{4}$ and $\frac{1}{6}$ is 12.
Find the number to multiply 4 to get 12.	$4 \cdot 3 = 12$
Find the number to multiply 6 to get 12.	$6 \cdot 2 = 12$
Use the Equivalent Fractions Property to convert each fraction to an equivalent fraction with the LCD,	$\frac{1}{4}$ $\frac{1}{6}$ $\frac{1 \cdot 3}{4 \cdot 3}$ $\frac{1 \cdot 2}{6 \cdot 2}$

multiplying both the numerator and denominator of each fraction by the same number.	
Simplify the numerators and denominators.	$\frac{3}{12}$ $\frac{2}{12}$

We do not reduce the resulting fractions. If we did, we would get back to our original fractions and lose the common denominator.

Note:
Exercise:

Problem: Change to equivalent fractions with the LCD:

$\frac{3}{4}$ and $\frac{5}{6}$, $\text{LCD} = 12$

Solution:

$\frac{9}{12}, \frac{10}{12}$

Note:
Exercise:

Problem: Change to equivalent fractions with the LCD:

$\frac{7}{12}$ and $\frac{11}{15}$, $\text{LCD} = 60$

Solution:

$\frac{35}{60}, \frac{44}{60}$

Note:
Convert two fractions to equivalent fractions with their LCD as the common denominator.

Find the LCD.
For each fraction, determine the number needed to multiply the denominator to get the LCD.
Use the Equivalent Fractions Property to multiply both the numerator and denominator by the number you found in Step 2.
Simplify the numerator and denominator.

Example:
Exercise:

Problem:

Convert $\frac{8}{15}$ and $\frac{11}{24}$ to equivalent fractions with denominator 120, their LCD.

Solution:
Solution

	The LCD is 120. We will start at Step 2.
Find the number that must multiply 15 to get 120.	$15 \cdot 8 = 120$
Find the number that must	$24 \cdot 5 = 120$

multiply 24 to get 120.	
Use the Equivalent Fractions Property.	$\frac{8 \cdot 8}{15 \cdot 8}$ $\frac{11 \cdot 5}{24 \cdot 5}$
Simplify the numerators and denominators.	$\frac{64}{120}$ $\frac{55}{120}$

Note:
Exercise:

Problem: Change to equivalent fractions with the LCD:

$\frac{13}{24}$ and $\frac{17}{32}$, LCD 96

Solution:

$\frac{52}{96}$, $\frac{51}{96}$

Note:
Exercise:

Problem: Change to equivalent fractions with the LCD:

$\frac{9}{28}$ and $\frac{27}{32}$, LCD 224

Solution:

$\frac{72}{224}, \frac{189}{224}$

Add and Subtract Fractions with Different Denominators

Once we have converted two fractions to equivalent forms with common denominators, we can add or subtract them by adding or subtracting the numerators.

Note:
Add or subtract fractions with different denominators.

Find the LCD.
Convert each fraction to an equivalent form with the LCD as the denominator.
Add or subtract the fractions.
Write the result in simplified form.

Example:
Exercise:

Problem: Add: $\frac{1}{2} + \frac{1}{3}$.

Solution:
Solution

	$\frac{1}{2} + \frac{1}{3}$
Find the LCD of 2, 3. 2, 4, 6 3, 6	LCD = 6
Change into equivalent fractions with the LCD 6.	$\frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2}$
Simplify the numerators and denominators.	$\frac{3}{6} + \frac{2}{6}$
Add.	$\frac{5}{6}$

Remember, always check to see if the answer can be simplified. Since 5 and 6 have no common factors, the fraction $\frac{5}{6}$ cannot be reduced.

You may want to do this problem with Fraction Towers to verify that the answer is correct.

Note:
Exercise:

Problem: Add: $\frac{1}{4} + \frac{1}{3}$.

Solution:

$\frac{7}{12}$

Note:
Exercise:

Problem: Add: $\frac{1}{2}+\frac{1}{5}$.

Solution:

$\frac{7}{10}$

Example:
Exercise:

Problem: Subtract: $\frac{1}{2}-\frac{1}{4}$.

Solution:
Solution

	$\frac{1}{2}-\frac{1}{4}$
Find the LCD of 2 and 4. 2, 4 4	LCD = 4
Rewrite as equivalent fractions using the LCD 4.	$\frac{1\cdot 2}{2\cdot 2}-\left(\frac{1}{4}\right)$

Simplify the first fraction.	$\frac{2}{4} - \frac{1}{4}$
Subtract.	$\frac{2-1}{4}$
Simplify.	$\frac{1}{4}$

One of the fractions already had the least common denominator, so we only had to convert the other fraction.

Note:
Exercise:

Problem: Simplify: $\frac{1}{2} - \frac{1}{8}$.

Solution:

$\frac{3}{8}$

Note:
Exercise:

Problem: Simplify: $\frac{1}{3} - \frac{1}{6}$.

Solution:

$\frac{1}{6}$

Note:

Exercise:

Problem: Add: $\frac{7}{12} + \frac{11}{15}$.

Solution:

$\frac{79}{60}$

Note:
Exercise:

Problem: Add: $\frac{13}{15} + \frac{17}{20}$.

Solution:

$\frac{103}{60}$

Note:
Exercise:

Problem: Subtract: $\frac{13}{24} - \frac{17}{32}$.

Solution:

$\frac{1}{96}$

Note:
Exercise:

Problem: Subtract: $\frac{21}{32} - \frac{9}{28}$.

Solution:

$\frac{75}{224}$

Key Concepts

- **Find the least common denominator (LCD) of two fractions.**

  Start a list of multiples for each denominator.
  Scan the list starting at the low end to see if they have a common multiple.
  The first common multiple found is the LCM.
  If no common multiple is found increase the list with the smallest last value.
  Check to see if the new value is a common multiple.
  If not repeat adding a value to the list with the smallest value and check again.

- **Equivalent Fractions Property**
  - If a, b, and c are whole numbers where $b \neq 0$, $c \neq 0$ then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$

- **Convert two fractions to equivalent fractions with their LCD as the common denominator.**

  Find the LCD.
  For each fraction, determine the number needed to multiply the denominator to get the LCD.
  Use the Equivalent Fractions Property to multiply both the numerator and denominator by the number you found in Step 2.
  Simplify the numerator and denominator.

- **Add or subtract fractions with different denominators.**

  Find the LCD.
  Convert each fraction to an equivalent form with the LCD as the denominator.
  Add or subtract the fractions.
  Write the result in simplified form.

- **Summary of Fraction Operations**

  - **Fraction addition:** Add the numerators and place the sum over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.
    Equation:

    $$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

  - **Fraction subtraction:** Subtract the numerators and place the difference over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.
    Equation:

    $$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Exercises

Practice Makes Perfect

Find the Least Common Denominator (LCD)

In the following exercises, find the least common denominator (LCD) for each set of fractions.
Exercise:

Problem: $\frac{2}{3}$ and $\frac{3}{4}$

Exercise:

Problem: $\frac{3}{4}$ and $\frac{2}{5}$

Solution:

20

Exercise:

Problem: $\frac{7}{12}$ and $\frac{5}{8}$

Exercise:

Problem: $\frac{9}{16}$ and $\frac{7}{12}$

Solution:

48

Exercise:

Problem: $\frac{13}{30}$ and $\frac{25}{42}$

Exercise:

Problem: $\frac{23}{30}$ and $\frac{5}{48}$

Solution:

240

Exercise:

Problem: $\frac{21}{35}$ and $\frac{39}{56}$

Exercise:

Problem: $\frac{18}{35}$ and $\frac{33}{49}$

Solution:

245

Exercise:

Problem: $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{3}{4}$

Exercise:

Problem: $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{3}{5}$

Solution:

60

Convert Fractions to Equivalent Fractions with the LCD

In the following exercises, convert to equivalent fractions using the LCD.

Exercise:

Problem: $\frac{1}{3}$ and $\frac{1}{4}$, $\text{LCD} = 12$

Exercise:

Problem: $\frac{1}{4}$ and $\frac{1}{5}$, $\text{LCD} = 20$

Solution:

$\frac{5}{20}, \frac{4}{20}$

Exercise:

Problem: $\frac{5}{12}$ and $\frac{7}{8}$, $\text{LCD} = 24$

Exercise:

Problem: $\frac{7}{12}$ and $\frac{5}{8}$, $\text{LCD} = 24$

Solution:

$\frac{14}{24}, \frac{15}{24}$

Exercise:

Problem: $\frac{13}{16}$ and $-\frac{11}{12}$, $\text{LCD} = 48$

Exercise:

Problem: $\frac{11}{16}$ and $-\frac{5}{12}$, $\text{LCD} = 48$

Solution:

$\frac{33}{48}, -\frac{20}{48}$

Exercise:

Problem: $\frac{1}{3}, \frac{5}{6}$, and $\frac{3}{4}$, $\text{LCD} = 12$

Exercise:

Problem: $\frac{1}{3}, \frac{3}{4}$, and $\frac{3}{5}$, $\text{LCD} = 60$

Solution:

$\frac{20}{60}, \frac{45}{60}, \frac{36}{60}$

Add and Subtract Fractions with Different Denominators

In the following exercises, add or subtract. Write the result in simplified form.

Exercise:

Problem: $\frac{1}{3} + \frac{1}{5}$

Exercise:

Problem: $\frac{1}{4} + \frac{1}{5}$

Solution:

$\frac{9}{20}$

Exercise:

Problem: $\frac{1}{2} + \frac{1}{7}$

Exercise:

Problem: $\frac{1}{3} + \frac{1}{8}$

Solution:

$\frac{11}{24}$

Exercise:

Problem: $\frac{2}{3} + \frac{3}{4}$

Exercise:

Problem: $\frac{3}{4} + \frac{2}{5}$

Solution:

$\frac{23}{20}$

Exercise:

Problem: $\frac{7}{12} + \frac{5}{8}$

Exercise:

Problem: $\frac{5}{12} + \frac{3}{8}$

Solution:

$\frac{19}{24}$

Exercise:

Problem: $\frac{7}{12} - \frac{9}{16}$

Exercise:

Problem: $\frac{7}{16} - \frac{5}{12}$

Solution:

$\frac{1}{48}$

Exercise:

Problem: $\frac{11}{12} - \frac{3}{8}$

Exercise:

Problem: $\frac{5}{8} - \frac{7}{12}$

Solution:

$\frac{1}{24}$

Exercise:

Problem: $\frac{2}{3} - \frac{3}{8}$

Exercise:

Problem: $\frac{5}{6} - \frac{3}{4}$

Solution:

$\frac{1}{12}$

Exercise:

Problem: $1 + \frac{7}{8}$

Exercise:

Problem: $1 + \frac{5}{6}$

Solution:

$\frac{11}{6}$

Exercise:

Problem: $1 - \frac{5}{9}$

Exercise:

Problem: $1 - \frac{3}{10}$

Solution:

$\frac{7}{10}$

Everyday Math

Exercise:

Problem:

Decorating Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{3}{16}$ yard of print fabric and $\frac{3}{8}$ yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?

Exercise:

Problem:

Baking Vanessa is baking chocolate chip cookies and oatmeal cookies. She needs $1\frac{1}{4}$ cups of sugar for the chocolate chip cookies, and $1\frac{1}{8}$ cups for the oatmeal cookies How much sugar does she need altogether?

Solution:

She needs $2\frac{3}{8}$ cups

Writing Exercises

Exercise:

Problem:

Explain why it is necessary to have a common denominator to add or subtract fractions.

Exercise:

Problem: Explain how to find the LCD of two fractions.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
add and subtract fractions with different denominators.			
identify and use fraction operations.			
use the order of operations to simplify complex fractions.			
evaluate variable expressions with fractions.			

ⓑ After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

Glossary

least common denominator (LCD)
: The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.

Multiply and Divide Fractions Beginning Level
By the end of this section, you will be able to:

- Multiply fractions
- Find reciprocals
- Divide fractions

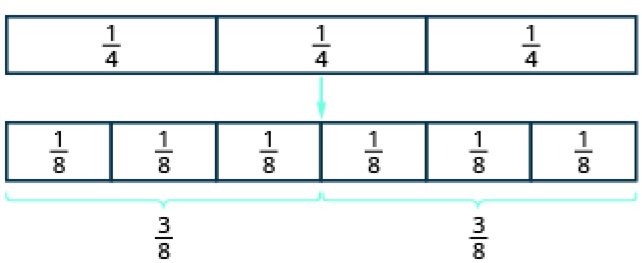
Note:
Before you get started, take this readiness quiz.

1. Draw a model of the fraction $\frac{3}{4}$.
   If you missed this problem, review [link].
2. Find two fractions equivalent to $\frac{5}{6}$.
   Answers may vary. Acceptable answers include $\frac{10}{12}$, $\frac{15}{18}$, $\frac{50}{60}$, etc.
   If you missed this problem, review [link].

Multiply fractions

A model may help you understand multiplication of fractions. We will use fraction tiles to model $\frac{1}{2} \cdot \frac{3}{4}$. To multiply $\frac{1}{2}$ and $\frac{3}{4}$, think $\frac{1}{2}$ of $\frac{3}{4}$.

Start with fraction tiles for three-fourths. To find one-half of three-fourths, we need to divide them into two equal groups. Since we cannot divide the three $\frac{1}{4}$ tiles evenly into two parts, we exchange them for smaller tiles.



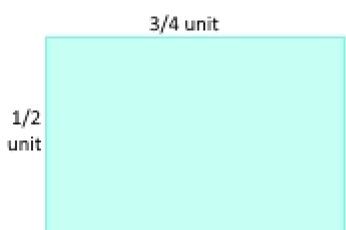
We see $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. Taking half of the six $\frac{1}{8}$ tiles gives us three $\frac{1}{8}$ tiles, which is $\frac{3}{8}$.

Therefore,

Equation:

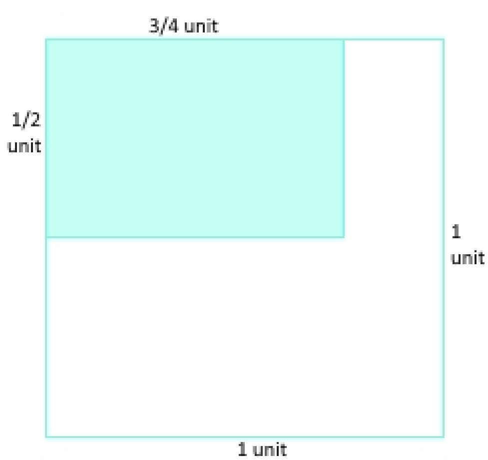
$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

The area model of multiplication can also be used to model this problem. We need a rectangle with one side $\frac{1}{2}$ of a unit long and the other side $\frac{3}{4}$ of a unit long.

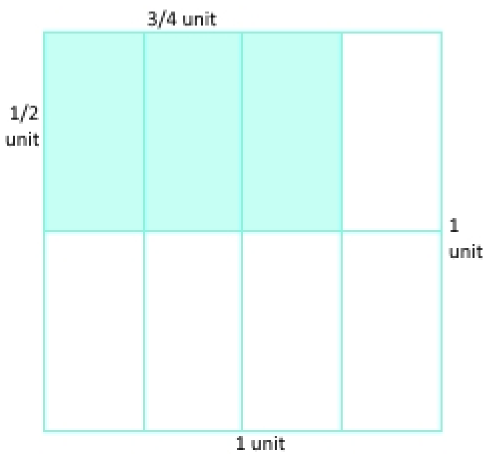


The area is the product of the sides.

We can find the area by extending the sides of this rectangle to make a unit square.



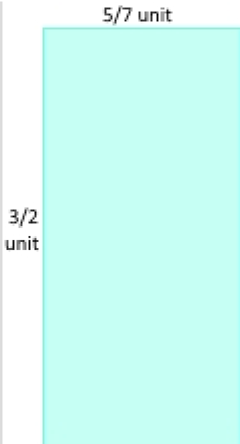
Draw in the lines that show the unit rectangle divided horizontally into half ($\frac{1}{2}$) and vertically into fourths ($\frac{1}{4}$).



All of the small rectangles are the same size. How many of them make up the unit square?
There are $2 \cdot 4 = 8$ of them; each one is $\frac{1}{8}$ of the unit square.

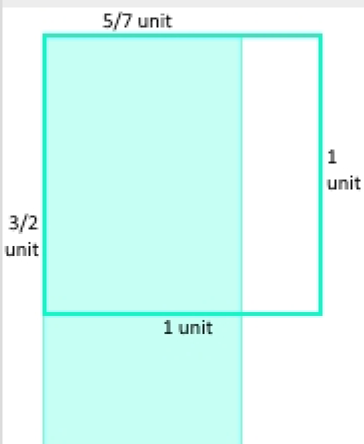
How many are shaded?
$1 \cdot 3 = 3$ of them. Therefore the area is $\frac{3}{8}$ of the square unit.

Note:
The area model of multiplication can also be used to model this problem. We need a rectangle with one side $\frac{3}{2}$ units long and the other side $\frac{5}{7}$ of a unit long.

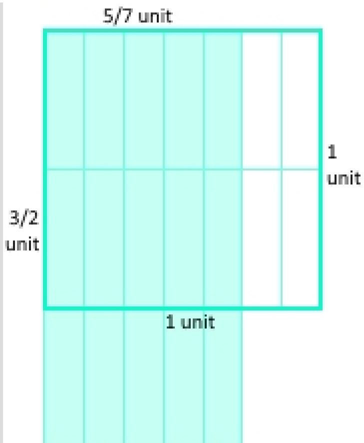


The area is the product of the sides.

We can find the area by extending the sides of this rectangle to make a unit square. The height is more than 1 unit long, so it is not fully included in the unit square.



Draw in the lines that show the unit rectangle divided horizontally into halves ($\frac{1}{2}$) and vertically into sevenths ($\frac{1}{7}$).



All of the small rectangles are the same size. How many of them make up the unit square?
There are $2 \cdot 7 = 14$ of them; each one is $\frac{1}{14}$ of the unit square.

How many are shaded?
$3 \cdot 5 = 15$ of them. Therefore the area is $\frac{15}{14}$ of the square unit. It's OK that some of these rectangles are outside of the unit square.
Exercise:

Problem: Use a diagram to model: $\frac{3}{4} \cdot \frac{5}{7}$.

Solution:

$\frac{15}{28}$

In the examples and exercises you may have noticed that we multiply the numerators of the factors to get the numerator of the product and multiply the denominators of the factors to get the denominator of the product. Normally, we then write the fraction in simplified form.

Note:
Fraction Multiplication
If $a, b, c,$ and d are numbers where $b \neq 0$ and $d \neq 0$, then
Equation:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Example:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{3}{4} \cdot \frac{1}{5}$.

Solution:
Solution

	$\frac{3}{4} \cdot \frac{1}{5}$
Multiply the numerators; multiply the denominators.	$\frac{3 \cdot 1}{4 \cdot 5}$
Simplify.	$\frac{3}{20}$

There are no common factors, so the fraction is simplified.

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{1}{3} \cdot \frac{2}{5}$.

Solution:

$\frac{2}{15}$

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{3}{5} \cdot \frac{7}{8}$.

Solution:

$\frac{21}{40}$

When multiplying fractions, the properties of positive and negative numbers still apply. It is a good idea to determine the sign of the product as the first step. In Example 4.26 we will multiply two negatives, so the product will be positive.

Example:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{5}{8}\left(\frac{2}{3}\right)$.

Solution:
Solution

	$\frac{5}{8}\left(\frac{2}{3}\right)$
Multiply the numerators, multiply the denominators.	$\frac{5\cdot 2}{8\cdot 3}$
Simplify.	$\frac{10}{24}$
Look for common factors in the numerator and denominator. Rewrite showing common factors.	$\frac{5\cdot\cancel{2}}{12\cdot\cancel{2}}$
Remove common factors.	$\frac{5}{12}$

Another way to find this product involves removing common factors earlier.

	$\frac{5}{8}\left(\frac{2}{3}\right)$
Multiply.	$\frac{5\cdot 2}{8\cdot 3}$
Show common factors and then remove them.	$\frac{5\cdot\cancel{2}}{4\cdot\cancel{2}\cdot 3}$
Multiply remaining factors.	$\frac{5}{12}$

We get the same result.

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{4}{7}\left(\frac{5}{8}\right)$.

Solution:

$\frac{5}{14}$

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{7}{12}\left(\frac{8}{9}\right)$.

Solution:

$\frac{14}{27}$

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{10}{28} \cdot \frac{8}{15}$.

Solution:

$\frac{4}{21}$

Note:

Exercise:

Problem: Multiply, and write the answer in simplified form: $\frac{9}{20} \cdot \frac{5}{12}$.

Solution:

$\frac{3}{16}$

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, $3 = \frac{3}{1}$, for example.

Example:
Exercise:

Problem: Multiply, and write the answer in simplified form:

$\frac{1}{7} \cdot 56$

Solution:
Solution

	$\frac{1}{7} \cdot 56$
Write 56 as a fraction.	$\frac{1}{7} \cdot \frac{56}{1}$
Determine the sign of the product; multiply.	$\frac{56}{7}$

Simplify.	8

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form:

ⓐ $\frac{1}{8} \cdot 72$
ⓑ $\frac{11}{3}(9)$

Solution:

ⓐ 9
ⓑ 33

Note:
Exercise:

Problem: Multiply, and write the answer in simplified form:

ⓐ $\frac{3}{8} \cdot 64$
ⓑ $16 \cdot \frac{11}{12}$

Solution:

ⓐ 24

ⓑ $\frac{44}{3}$

Find Reciprocals

The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are related to each other in a special way. So are $\frac{10}{7}$ and $\frac{7}{10}$. Do you see how? Besides looking like upside-down versions of one another, if we were to multiply these pairs of fractions, the product would be 1.

Equation:

$$\frac{2}{3} \cdot \frac{3}{2} = 1 \quad \text{and} \quad \frac{10}{7}\left(\frac{7}{10}\right) = 1$$

Such pairs of numbers are called reciprocals.

Note:

Reciprocal

The **reciprocal** of the fraction $\frac{a}{b}$ is $\frac{b}{a}$, where $a \neq 0$ and $b \neq 0$,

A number and its reciprocal have a product of 1. Notice that if X is the reciprocal of Y then Y is the reciprocal of X.

Equation:

$$\frac{a}{b} \cdot \frac{b}{a} = 1$$

To find the reciprocal of a fraction, we invert the fraction. This means that we place the numerator in the denominator and the denominator in the numerator.

The number zero does not have a reciprocal. Why? A number and its reciprocal multiply to 1. Is there any number r so that $0 \cdot r = 1$? No. So, the number 0 does not have a reciprocal.
If we were to invert a fraction with zero in the numerator then new fraction would have zero in the denominator. But division by zero is undefined.

Example:
Exercise:

Problem:

Find the reciprocal of each number. Then check that the product of each number and its reciprocal is 1.

ⓐ $\frac{4}{9}$
ⓑ $\frac{1}{6}$
ⓒ $\frac{14}{5}$
ⓓ 7

Solution:
Solution

To find the reciprocals, we keep the sign and invert the fractions.

ⓐ	
Find the reciprocal of $\frac{4}{9}$.	The reciprocal of $\frac{4}{9}$ is $\frac{9}{4}$.
Check:	

Multiply the number and its reciprocal.	$\frac{4}{9} \cdot \frac{9}{4}$
Multiply numerators and denominators.	$\frac{36}{36}$
Simplify.	1

ⓑ	
Find the reciprocal of $\frac{1}{6}$.	$\frac{6}{1}$
Simplify.	6
Check:	$\frac{1}{6} \cdot (6)$
	1

ⓒ	
Find the reciprocal of $\frac{14}{5}$.	$\frac{5}{14}$
Check:	$\frac{14}{5}\left(\frac{5}{14}\right)$

	$\frac{70}{70}$
	1

ⓓ	
Find the reciprocal of 7.	
Write 7 as a fraction.	$\frac{7}{1}$
Write the reciprocal of $\frac{7}{1}$.	$\frac{1}{7}$
Check:	$7 \cdot \left(\frac{1}{7}\right)$
	1

Note:
Exercise:

Problem: Find the reciprocal:

ⓐ $\frac{5}{7}$
ⓑ $\frac{1}{8}$
ⓒ $\frac{11}{4}$
ⓓ 14

Solution:

ⓐ $\frac{7}{5}$
ⓑ 8
ⓒ $\frac{4}{11}$
ⓓ $\frac{1}{14}$

Note:
Exercise:

Problem: Find the reciprocal:

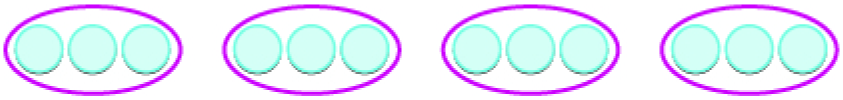
ⓐ $\frac{3}{7}$
ⓑ $\frac{1}{12}$
ⓒ $\frac{14}{9}$
ⓓ 21

Solution:

ⓐ $\frac{7}{3}$
ⓑ 12
ⓒ $\frac{9}{14}$
ⓓ $\frac{1}{21}$

Divide Fractions

Why is $12 \div 3 = 4$? We previously modeled this with counters. How many groups of 3 counters can be made from a group of 12 counters?



There are 4 groups of 3 counters. In other words, there are four 3s in 12.
So, $12 \div 3 = 4$.
For every division there is a related multiplication. In this case, $4 \cdot 3 = 12$.

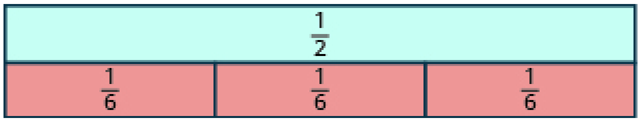
Division can be thought of as subtraction where the same number is subtracted over and over again. How many times can 3 be subtracted from 12?
$12 - 3 = 9$. $9 - 3 = 6$. $6 - 3 = 3$. $3 - 3 = 0$.
This also gives the answer 4.

Dividing fractions is similar. Suppose we want to find the quotient: $\frac{1}{2} \div \frac{1}{6}$. We need to figure out how many $\frac{1}{6}$ there are in $\frac{1}{2}$. We can use fraction tiles to model this division. We start by lining up the half and sixth fraction tiles as shown in the next figure. Notice, there are three $\frac{1}{6}$ tiles in $\frac{1}{2}$, so $\frac{1}{2} \div \frac{1}{6} = 3$.The related multiplication problem is $3 \cdot \frac{1}{6} = \frac{1}{2}$.

Thought of as repeated subtraction, the first step is: $\frac{1}{2} - \frac{1}{6}$.
This requires a common denominator which is 6. The problem now becomes:
$\frac{3}{6} - \frac{1}{6} = \frac{2}{6}$.

$\frac{2}{6} - \frac{1}{6} = \frac{1}{6}$.

$\frac{1}{6} - \frac{1}{6} = 0$.
This also gives the answer 3.

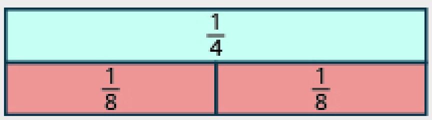


Example:
Exercise:

Problem: Model: $\frac{1}{4} \div \frac{1}{8}$. Show the corresponding multiplication.

Solution:
Solution

We want to determine how many $\frac{1}{8}$ are in $\frac{1}{4}$. Start with one $\frac{1}{4}$ tile. Line up $\frac{1}{8}$ tiles underneath the $\frac{1}{4}$ tile.



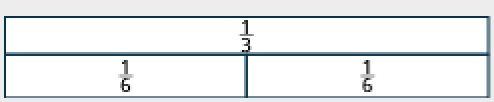
There are two $\frac{1}{8}$ in $\frac{1}{4}$.

So, $\frac{1}{4} \div \frac{1}{8} = 2$. The corresponding multiplication is $2 \cdot \frac{1}{8} = \frac{1}{4}$.

Note:
Exercise:

Problem: Model: $\frac{1}{3} \div \frac{1}{6}$. Show the corresponding multiplication.

Solution:

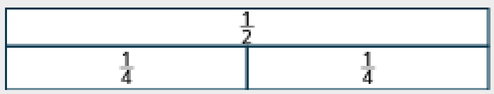


The corresponding multiplication is $2 \cdot \frac{1}{6} = \frac{1}{3}$.

Note:
Exercise:

Problem: Model: $\frac{1}{2} \div \frac{1}{4}$. Show the corresponding multiplication.

Solution:



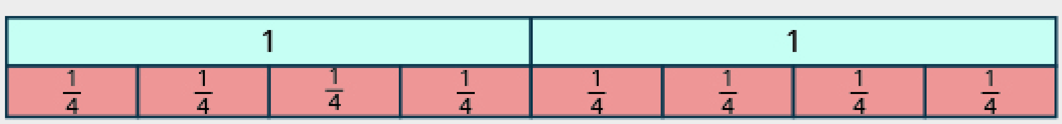
The corresponding multiplication is $2 \cdot \frac{1}{4} = \frac{1}{2}$.

Example:
Exercise:

Problem: Model: $2 \div \frac{1}{4}$. Show the corresponding multiplication.

Solution:
Solution

We are trying to determine how many $\frac{1}{4}$ there are in 2. We can model this as shown.

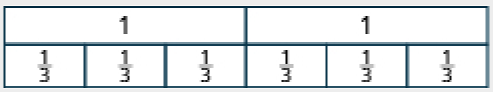


There are eight $\frac{1}{4}$ in 2.
$2 \div \frac{1}{4} = 8$.
The corresponding multiplication is $8 \cdot \frac{1}{4}$ =2.

Note:
Exercise:

Problem: Model: $2 \div \frac{1}{3}$ Show the corresponding multiplication.

Solution:

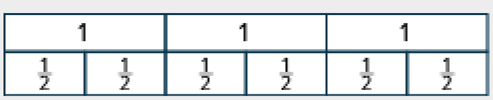


The corresponding multiplication is $6 \cdot \frac{1}{3} = 2$.

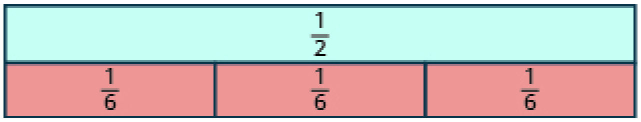
Note:
Exercise:

Problem: Model: $3 \div \frac{1}{2}$ Show the corresponding multiplication.

Solution:



The corresponding multiplication is $6 \cdot \frac{1}{2} = 3$.

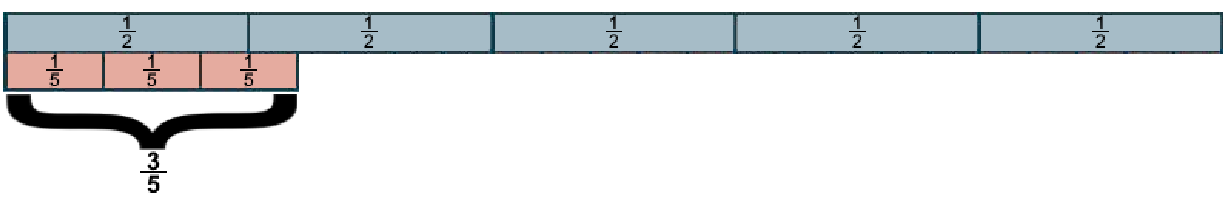


Recall $\frac{1}{2} \div \frac{1}{6}$.
Assume that we divided $\frac{1}{2}$ a foot by $\frac{1}{6}$ of a foot. The answer is still 3, not 3 feet but just the number 3. We can divide a $\frac{1}{2}$ foot long strip into 3 $\frac{1}{6}$ of a foot long strips. This is true of any other unit such as miles, pounds, or gallons. For example, if I have $\frac{1}{2}$ a pound of candy then I can divide it into 3 $\frac{1}{6}$ pound portions. The unit we use to measure doesn't matter as long as both parts are measured using the same unit.

Imagine if we measured in inches rather than in feet. 12 inches = 1 foot so $\frac{1}{2}$ foot = 6 inches and $\frac{1}{6}$ foot = 2 inches.
What is 6 inches divided by 2 inches? $6 \div 2 = 3$. This shows we can easily divide fractions if we can change the unit to get rid of the fractions. We are then left with a problem that just divides whole numbers.

The close relationship between division and subtraction means that we can use our knowledge of subtraction to help us with more difficult divisions. So far, our division of fraction problems have been easy. We've been able to illustrate them with tiles and they have worked out evenly. Let's try a harder problem: $\frac{5}{2} \div \frac{3}{5}$.

How much is left after we subtract: $\frac{5}{2} - \frac{3}{5}$?



In order to do the subtraction we need to find a common denominator. In this case, $2 \cdot 5 = 10$.

The next step is to change each fraction to an equivalent fraction with a denominator of 10.

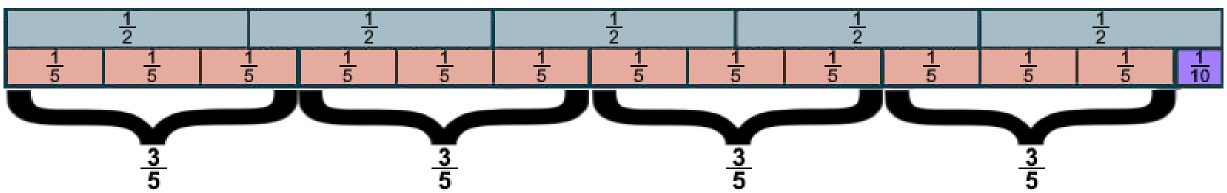
$\frac{5}{2} = \frac{5 \cdot 5}{2 \cdot 5} = \frac{25}{10}$ and $\frac{3}{5} = \frac{3 \cdot 2}{5 \cdot 2} = \frac{6}{10}$.

Subtracting $\frac{25}{10} - \frac{6}{10} = \frac{19}{10}$.

Again $\frac{19}{10} - \frac{6}{10} = \frac{13}{10}$.

Again $\frac{13}{10} - \frac{6}{10} = \frac{7}{10}$.

Again $\frac{7}{10} - \frac{6}{10} = \frac{1}{10}$ until we can no longer take away $\frac{6}{10}$.



We were able to subtract $\frac{3}{5}$ from $\frac{5}{2}$ 4 times and $\frac{1}{10}$ was left over. The 4 is easy to understand, but what about the part that remains? It is $\frac{1}{10}$ of a unit, but what we want to know is what part of $\frac{3}{5}$ it is.

Recall that $\frac{3}{5}$ is equivalent to $\frac{6}{10}$.

Thought of that way, the individual pieces are the same size, they are both $\frac{1}{10}$. We can easily tell that it takes 6 $\frac{1}{10}$ pieces to make $\frac{6}{10}$ so $\frac{1}{10}$ is $\frac{1}{6}$ of $\frac{6}{10}$. Therefore the final answer is 4 $\frac{1}{6}$.

Caution: 4 $\frac{1}{10}$ is not the answer. The $\frac{1}{10}$ is the size of the piece remaining, but not what part of $\frac{3}{5}$ that piece is.

If you are thinking that there must be an easier way, don't worry there is. Just like you wouldn't normally want to multiply fractions by repeated addition, you don't normally want to divide fractions by repeated subtraction. The important part is to notice that changing to a common denominator was an essential part of the solution.

Once we had changed to $\frac{25}{10}$ divided by $\frac{6}{10}$ we were almost done. Imagine that we had measured $\frac{25}{10}$ and $\frac{6}{10}$ in units that were $\frac{1}{10}$ the size. Then the amounts would have been 25 and 6 and we'd be dividing with whole numbers. Since the unit of measurement does not matter as long as it is the same for both parts, our answer would be $\frac{25}{6} = 4\frac{1}{6}$.

This agrees with the Equivalent Fraction Property.

If a, b, and c are numbers where $b \neq 0$ and $c \neq 0$, then
Equation:

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

Previously, we had thought of c as an integer, $c \neq 0$, but it can also be a fraction. Also remember that an equation can be used in either direction.
Equation:

Division of Fractions with a Common Denominator

$$\frac{a}{c} \div \frac{b}{c} = \frac{a}{b}$$

From the example above,
Equation:

$$\frac{25}{10} \div \frac{6}{10} = \frac{25}{6}$$

Dividing by 10 is the same as multiplying by $\frac{1}{10}$. In doing division of fractions, we are using the Equivalent Fraction Property going from the right side of the property to the left side. Once we have a common denominator, we can immediately write $\frac{25}{6}$.

An Intuitive Look at the Equivalent Fraction Property

How many 3 pound portions do I get when I divide 12 pounds of food? $12 \div 3 = 4$.

What if I make both the portion size and total amount of food twice as big? How many $3 \cdot 2 = 6$ pound portions do I get when I divide $12 \cdot 2 = 24$ pounds of food? $24 \div 6 = 4$.

What if I make both the portion size and total amount of food ten times as big?
How many $3 \cdot 10 = 30$ pound portions do I get when I divide $12 \cdot 10 = 120$ pounds of food? $120 \div 30 = 4$.

In each case the answer is 4 portions. Why? In each case the numerator and denominator were multiplied by the same factor. Therefore the Equivalent Fractions Property applies.

Make the portion size and the amount of food smaller by the same factor.

What if I make both the portion size and total amount of food half as big? How many $3 \div 2 = \frac{3}{2}$ pound portions do I get when I divide $12 \div 2 = \frac{12}{2}$ pounds of food? $\frac{12}{2} \div \frac{3}{2} = 4$.

What if I make both the portion size and total amount of food ten times smaller?
How many $3 \div 10 = \frac{3}{10}$ pound portions do I get when I divide $12 \div 10 = \frac{12}{10}$ pounds of food? $\frac{12}{10} \div \frac{3}{10} = 4$.

In each case the answer is 4 portions. Why? In each case, the numerator and denominator were divided by the same factor. Dividing by a factor can be thought of as multiplying by 1 divided by the same factor.

Sometimes it is easier to see the math idea when we think of money.

How many $\$3$ candies can I buy if I have 15 one dollar bills? $\$15 \div \$3 = 5$

.
How many 75 cents = 3 quarters candies can I buy if I have 15 quarters? $\frac{15}{4}$ of a dollar $\div \frac{3}{4}$ of a dollar $= 5$.
How many 30 cents = 3 dime candies can I buy if I have 15 dimes? $\frac{15}{10}$ of a dollar $\div \frac{3}{10}$ of a dollar $= 5$.

Since the denominators are the same, the division of fractions was easy. We just divided the numerators.

Dividing Fractions with Different Denominators
Just as with adding and subtracting fractions with different denominators the first step is to get a common denominator.
It is not important if the denominator is the LCM, just as long as it is a common multiple. The fraction will normally be simplified at the end of the problem.

Example:
$\frac{2}{3} \div \frac{4}{5}$

A common denominator for 3 and 5 is 15. Change each fraction to equivalent fractions with a denominator of 15.

$\frac{2 \cdot 5}{3 \cdot 5} \div \frac{4 \cdot 3}{5 \cdot 3}$

By the commutative property $3 \cdot 5 = 5 \cdot 3$. We don't have to do the extra work of seeing that is 15 (although we already did.)
Therefore the fractions have the same denominator and the problem simplifies to $\frac{2 \cdot 5}{4 \cdot 3}$. This can be simplified to $\frac{5}{6}$.

Note:
Dividing Fractions Using the Reciprocal
If you previously learned how to divide fractions it is probably by multiplying by the reciprocal of the divisor.
If $a, b, c,$ and d are numbers where $b \neq 0, c \neq 0,$ and $d \neq 0$, then

Equation:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

We need to say $b \neq 0, c \neq 0$ and $d \neq 0$ to be sure we don't divide by zero. To divide fractions, multiply the first fraction by the reciprocal of the second.

Why does this work?

Multiplying by the reciprocal is equivalent to finding a common denominator and simplifying.

Doing the multiplication: $\frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{c \cdot b}$, which is exactly the result from the common denominator method.

There is very little difference between the two methods and you can use the one you prefer now that you understand why they work.

Example:

Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{2}{5} \div \frac{3}{7}$.

Solution:

Solution

	$\frac{2}{5} \div \frac{3}{7}$
Multiply the first fraction by the reciprocal of the second.	$\frac{2}{5} \cdot \frac{7}{3}$
Multiply.	$\frac{14}{15}$

Example:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{2}{3} \div \frac{}{5}$.

Solution:
Solution

	$\frac{2}{3} \div \frac{}{5}$
Multiply the first fraction by the reciprocal of the second.	$\frac{2}{3} \cdot \frac{5}{}$
Multiply.	$\frac{10}{21}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{3}{7} \div \frac{2}{3}$.

Solution:

$\frac{9}{14}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{2}{3} \div \frac{7}{9}$.

Solution:

$\frac{6}{7}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{3}{5} \div \frac{21}{10}$.

Solution:

$\frac{2}{7}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{7}{27} \div \frac{35}{36}$.

Solution:

$\frac{4}{15}$

Note:
Exercise:

Problem: Divide, and write the answer in simplified form: $\frac{5}{14} \div \frac{15}{28}$.

Solution:

$\frac{2}{3}$

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Multiplying Fractions
- Dividing Fractions

Key Concepts

- **Equivalent Fractions Property**
  - If a, b, c are numbers where $b \neq 0$, $c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.
- **Simplify a fraction.**

  Rewrite the numerator and denominator to show the common factors.
  Simplify, using the equivalent fractions property, by removing common factors.
  Multiply any remaining factors.
- **Fraction Multiplication**

- If a, b, c, and d are numbers where $b \neq 0$and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

- **Reciprocal**
  - A number and its reciprocal have a product of 1.

- **Fraction Division**
  - If a, b, c, and d are numbers where $b \neq 0$, $c \neq 0$ and $d \neq 0$, then **Equation:**

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

  - To divide fractions, multiply the first fraction by the reciprocal of the second.

Exercises

Practice Makes Perfect

Multiply Fractions

In the following exercises, use a diagram to model.
Exercise:

Problem: $\frac{1}{2} \cdot \frac{2}{3}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{1}{2} \cdot \frac{5}{8}$

Exercise:

Problem: $\frac{1}{3} \cdot \frac{5}{6}$

Solution:

$\frac{5}{18}$

Exercise:

Problem: $\frac{1}{3} \cdot \frac{2}{5}$

In the following exercises, multiply, and write the answer in simplified form.

Exercise:

Problem: $\frac{2}{5} \cdot \frac{1}{3}$

Solution:

$\frac{2}{15}$

Exercise:

Problem: $\frac{1}{2} \cdot \frac{3}{8}$

Exercise:

Problem: $\frac{3}{4} \cdot \frac{9}{10}$

Solution:

$\frac{27}{40}$

Exercise:

Problem: $\frac{4}{5} \cdot \frac{2}{7}$

Exercise:

Problem: $\frac{2}{3}\left(\frac{3}{8}\right)$

Solution:

$\frac{1}{4}$

Exercise:

Problem: $\frac{3}{4}\left(\frac{4}{9}\right)$

Exercise:

Problem: $\frac{5}{9} \cdot \frac{3}{10}$

Solution:

$\frac{1}{6}$

Exercise:

Problem: $\frac{3}{8} \cdot \frac{4}{15}$

Exercise:

Problem: $\frac{7}{12}\left(\frac{8}{21}\right)$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $\frac{5}{12}\left(\frac{8}{15}\right)$

Exercise:

Problem: $\left(\frac{14}{15}\right)\left(\frac{9}{20}\right)$

Solution:

$\frac{21}{50}$

Exercise:

Problem: $\left(\frac{9}{10}\right)\left(\frac{25}{33}\right)$

Exercise:

Problem: $\left(\frac{63}{84}\right)\left(\frac{44}{90}\right)$

Solution:

$\frac{11}{30}$

Exercise:

Problem: $\left(\frac{33}{60}\right)\left(\frac{40}{88}\right)$

Exercise:

Problem: $4 \cdot \frac{5}{11}$

Solution:

$\frac{20}{11}$

Exercise:

Problem: $5 \cdot \frac{8}{3}$

Exercise:

Problem: $\frac{3}{7} \cdot 21$

Solution:

9

Exercise:

Problem: $\frac{5}{6} \cdot 30$

Exercise:

Problem: $28\left(\frac{1}{4}\right)$

Solution:

7

Exercise:

Problem: $51\left(\frac{1}{3}\right)$

Exercise:

Problem: $8\left(\frac{17}{4}\right)$

Solution:

34

Exercise:

Problem: $\frac{14}{5}(15)$

Exercise:

Problem: $1\left(\frac{3}{8}\right)$

Solution:

$\frac{3}{8}$

Exercise:

Problem: $(1)\left(\frac{6}{7}\right)$

Find Reciprocals

In the following exercises, find the reciprocal.

Exercise:

Problem: $\frac{3}{4}$

Solution:

$\frac{4}{3}$

Exercise:

Problem: $\frac{2}{3}$

Exercise:

Problem: $\frac{5}{17}$

Solution:

$\frac{17}{5}$

Exercise:

Problem: $\frac{6}{19}$

Exercise:

Problem: $\frac{11}{8}$

Solution:

$\frac{8}{11}$

Exercise:

Problem: 13

Exercise:

Problem: 19

Solution:

$\frac{1}{19}$

Exercise:

Problem: 1

Solution:

1

Divide Fractions

In the following exercises, model each fraction division.

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{4}$

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{8}$

Solution:

4

Exercise:

Problem: $2 \div \frac{1}{5}$

Exercise:

Problem: $3 \div \frac{1}{4}$

Solution:

12

In the following exercises, divide, and write the answer in simplified form.

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{4}$

Exercise:

Problem: $\frac{1}{2} \div \frac{1}{8}$

Solution:

4

Exercise:

Problem: $\frac{3}{4} \div \frac{2}{3}$

Exercise:

Problem: $\frac{4}{5} \div \frac{3}{4}$

Solution:

$\frac{16}{15}$

Exercise:

Problem: $\frac{4}{5} \div \frac{4}{7}$

Exercise:

Problem: $\frac{3}{4} \div \frac{3}{5}$

Solution:

$\frac{5}{4}$

Exercise:

Problem: $\frac{7}{9} \div \left(\frac{7}{9}\right)$

Exercise:

Problem: $\frac{5}{6} \div \left(\frac{5}{6}\right)$

Solution:

1

Exercise:

Problem: $\frac{5}{18} \div \left(\frac{15}{24}\right)$

Exercise:

Problem: $\frac{7}{18} \div \left(\frac{14}{27}\right)$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{7}{12} \div \frac{21}{8}$

Exercise:

Problem: $\frac{5}{12} \div \frac{15}{8}$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $5 \div \frac{1}{2}$

Exercise:

Problem: $3 \div \frac{1}{4}$

Solution:

12

Exercise:

Problem: $\frac{3}{4} \div (12)$

Exercise:

Problem: $\frac{2}{5} \div (10)$

Solution:

$\frac{1}{25}$

Exercise:

Problem: $18 \div \left(\frac{9}{2}\right)$

Exercise:

Problem: $15 \div \left(\frac{5}{3}\right)$

Solution:

9

Exercise:

Problem: $\frac{1}{2} \div \left(\frac{3}{4}\right) \div \frac{7}{8}$

Exercise:

Problem: $\frac{11}{2} \div \frac{7}{8} \cdot \frac{2}{11}$

Solution:

$\frac{8}{7}$

Everyday Math

Exercise:

Problem:

Baking A recipe for chocolate chip cookies calls for $\frac{3}{4}$ cup brown sugar. Imelda wants to double the recipe.

ⓐ How much brown sugar will Imelda need? Show your calculation. Write your result as an improper fraction and as a mixed number.

ⓑ Measuring cups usually come in sets of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the recipe.

Exercise:

Problem:

Baking Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs $\frac{2}{3}$ cup of condensed milk.

ⓐ How much condensed milk will Nina need? Show your calculation. Write your result as an improper fraction and as a mixed number.
ⓑ Measuring cups usually come in sets of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Nina could measure the condensed milk she needs.

Solution:

- ⓐ $4\frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$
- ⓑ Answers will vary.

Exercise:

Problem:

Portions Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?

Exercise:

Problem:

Portions Kristen has $\frac{3}{4}$ yards of ribbon. She wants to cut it into equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Solution:

$\frac{1}{8}\text{ yard}$

Writing Exercises

Exercise:

Problem: Explain how you find the reciprocal of a fraction.

Exercise:

Problem:

Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 or 8 slices. Would he prefer 3 out of 6 slices or 4 out of 8 slices? Rafael replied that since he wasn't very hungry, he would prefer 3 out of 6 slices. Explain what is wrong with Rafael's reasoning.

Exercise:

Problem:

Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify fractions.			
multiply fractions.			
find reciprocals.			
divide fractions.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

reciprocal
: The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$ where $a \neq 0$ and $b \neq 0$.